Logical derivations using sequent calculi: Lab2

October 27, 2006

- Web site of the module:

  http://www.macs.hw.ac.uk/∼ lilia/ar/ar.html

- Web site of PVS:

  http://pvs.csl.sri.com/

- PVS documentation:

  http://pvs.csl.sri.com/documentation.shtml

- Task 1: Examine the sequents, written in PVS syntax in the following PVS Example File. Write the sequents in first-order logic.

```
seq_xmpls: THEORY

BEGIN

X, Y: TYPE
A, B, C, D: bool
P, Q, T: PRED[X]
R: [X, Y -> bool]

seq1: THEOREM A OR (A IMPLIES B)

seq2: THEOREM (A IMPLIES B) IFF (NOT A OR B)

seq3: THEOREM ((A AND B) OR (C AND D)) IMPLIES ((A OR C) AND (B OR D))

seq4: THEOREM (EXISTS (x:X): NOT P(x)) OR (FORALL (x:X): P(x))

seq5: THEOREM (EXISTS (y:Y): FORALL (x:X): R(x,y)) IMPLIES
        (FORALL (x:X): EXISTS (y:Y): R(x,y))

seq6: THEOREM ((FORALL (x:X): P(x)) OR (FORALL (x:X): Q(x))) IMPLIES
        (FORALL (x:X): P(x) OR Q(x))
```

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seq7: THEOREM (FORALL (x,y:X): P(x) OR Q(y)) OR (FORALL (y,z:X): Q(y) OR
T(z))
         IMPLIES (FORALL (x,y,z:X): P(x) OR Q(y) OR T(z))

END seq_xmpls
```

- Task 2: Prove the following sequences in PVS

$$
\begin{array}{ll}
(1) & \vdash \neg\neg A \to A \\
(2) & \vdash \neg\neg(\neg\neg A \to A) \\
(3) & \vdash A \vee B \to (A \to C) \to (B \to C) \to C \\
(4) & \vdash A \to B \to (A \wedge B) \\
(5) & \vdash A \to A \vee B \\
(6) & \forall x \forall y A(x,y) \vdash \forall y \forall x A(x,y) \\
(7) & \exists x (A(x) \to B) \vdash (\forall x A(x) \to (B)) \\
(8) & \neg \exists x A(x) \vdash \forall x \neg A(a) \\
(9) & (\exists x (A(x) \vee B(x))) \vdash (\exists x A(x) \vee \exists x (B(x))) \\
(10) & \forall x (A(x) \to B) \vdash (\forall x A(x) \to B)
\end{array}
$$