# Coursework 1: Automated Reasoning for Software Engineering

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This piece of coursework is due by November 18th, 2005.

1.A. Formalize the following statements in the signature $\Sigma_{PA}$ of Peano arithmetic.

1. 3 is not divisible by 2. (0.5 points)
2. All numbers between 1 and 3 are even. (0.5 points)
3. There exists exactly one number between 1 and 3. (1 point)
4. There does not exist the largest square number. (1 point)

Total for part 1A: 3 points.

1. B. Compute the result of the following substitutions:

1. $\forall y(p(f(y,x)))[x/y]$
2. $\forall y(p(f(y,x)))[y/x]$
3. $\forall y(p(f(z,g(y))) \vee \exists z(g(z) \approx y))[g(b)/z]$
4. $\exists y(f(x,y) \approx x \rightarrow \forall x(f(x,y) \approx x))[g(y)/y, g(z)/x]$

Total for part 1B: 4 points: one for each correct answer.

2. A. Give proofs for the following formulae in sequent calculi.

1. $A \vee (A \rightarrow C)$ (0.5 points)
2. $(A \rightarrow C) \leftrightarrow (\neg A \vee C)$ (0.5 points)
3. $((A \wedge B) \vee (C \wedge D)) \rightarrow ((A \vee C) \wedge (B \vee D))$ (1 point)
4. $\exists x : X \neg P(x) \vee \forall x : X P(x)$ (1 point)
5. $(\exists y : Y \forall x : X P(x,y)) \rightarrow (\forall x : X \exists y : Y P(x,y))$ (1 point)
6. $(\forall x : X P(x) \vee \forall x : X Q(x)) \rightarrow (\forall x : X P(x) \vee Q(x))$ (1 point)

7. $(\forall x, y : X P(x) \vee Q(y)) \vee (\forall y, z : X Q(y) \vee T(z)) \to (\forall x, y, z : X P(x) \vee Q(y) \vee T(z))$ (1 point)

Total for part A: 6 Points.

B. Use PVS to prove the formulae 1–7. Use only the basic commands *case, flatten, split, propax, instantiate* and *skolem*. These are the commands that implement the sequent calculus in PVS (See Chapter 3 of the PVS prover guide). For each formula provide the commands that you have used to prove it.

Total for part B: 7 points (one point for each proof).