Coursework 1: Automated Reasoning and SE

Lilia Georgieva

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This piece of coursework is due on December, 10th., 2004.

1. A. Give proofs for the following formulae in sequent calculi.

1. $A \vee (A \rightarrow C)$ (0.5 points)
2. $(A \rightarrow C) \leftrightarrow (\neg A \vee C)$ (0.5 points)
3. $((A \wedge B) \vee (C \wedge D)) \rightarrow ((A \vee C) \wedge (B \vee D))$ (1 point)
4. $\exists x : X \neg P(x) \vee \forall x : X P(x)$ (1 point)
5. $(\exists y : Y \forall x : X P(x, y)) \rightarrow (\forall x : X \exists y : Y P(x, y))$ (1 point)
6. $(\forall x : X P(x) \vee \forall x : X Q(x)) \rightarrow (\forall x : X P(x) \vee Q(x))$ (1 point)
7. $(\forall x, y : X P(x) \vee Q(y)) \vee (\forall y, z : X Q(y) \vee T(z)) \rightarrow (\forall x, y, z : X P(x) \vee Q(y) \vee T(z))$ (1 point)

Total for part A: 6 Points.

B. Use PVS to prove the formulae 1–7. Use only the basic commands *case, split, propax, instantiate* and *skolem*. These are the commands that implement the sequent calculus in PVS (See Chapter 3 of the PVS prover guide). For each formula provide the commands that you have used to prove it.

Total for part B: 7 points (one point for each proof).

2. A. Which of the following pairs of terms are α or β or η equivalent?

1. $\lambda x : X P(x)$ and $\lambda y : X Q(x)$
2. $\lambda x : N \lambda y : N p(x, y)$ and $\lambda y : N \lambda x : N p(y, x)$
3. $\lambda p(X \rightarrow Form) \lambda x : X p(x)$ and $\lambda q(X \rightarrow Form) \lambda y : X q(y)$
4. $\lambda x : X f(x)$ and $(\lambda y : (X \rightarrow X) y).f$

B. Substitute and then β reduce:

1. $\lambda x : N p(x) \wedge q(x)$ for p in $\forall x : N p(x)$
2. $\lambda x : N \exists y p(x, y)$ for p in $\forall x : N p(f(y))$
3. $\lambda x, y : N x = y$ for p in $\forall x : N \exists y : N p(x, y)$

Total for question 2: 7 points (one point for each subproblem).