Logical derivation using sequent calculi: Lab2

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- Web site of the module:

  http://www.macs.hw.ac.uk/$\sim$ lilia/ar/ar.html

- Web site of PVS:

  http://pvs.csl.sri.com/

- PVS documentation:

  http://pvs.csl.sri.com/documentation.shtml

- PVS Examples:

  http://www.macs.hw.ac.uk/ $\sim$ lilia/pvs/exercises/

- Give derivations of the following sequences, indicating clearly which rules you have used:

$$\begin{array}{ll} \vdash & \neg\neg A \to A \\ \vdash & \neg\neg(\neg\neg A \to A \\ \vdash & A \vee B \to (A \to C) \to (B \to C) \to C \\ \vdash & A \to B \to (A \wedge B) \\ \vdash & A \to A \vee B \end{array}$$

- For first-order propositional sequent calculi in PVS *flatten* corresponding to the rules of the sequent calculi where reading upwards a sequent is transformed into one sequent and *split* where reading upwards a sequent is transformed into two sequents.

  First prove by hand and then use PVS to derive:

$$\begin{array}{ll} \vdash & \neg\neg A \to A \\ \vdash & \neg\neg(\neg\neg A \to A \\ \vdash & A \vee B \to (A \to C) \to (B \to C) \to C \\ \vdash & A \to B \to (A \wedge B) \\ \vdash & A \to A \vee B \end{array}$$

  Note: it is OK to use two steps where PVS uses one provided that the steps are correct.

- Peano arithmetic (pointers for solving the exercise in your coursework)

  http://en.wikipedia.org/wiki/Peano_axioms

  Reminder: Peano arithmetic is used to define properties of natural (non-negative) numbers and uses 0 and the successor function only.

  Examples:

  Definition of + using the axioms:

$$\forall x (0 + x) = x$$

$$\forall x \forall y (x + s(y)) = s(x + y)$$

  How do we prove that $1 + 1 = 2$ in Peano arithmetic:

$$\begin{aligned} 1 + 1 & = \\ & = s(0) + s(0) = \\ & = s(0 + s(0)) = \\ & = s(s(0)) = \\ & = 2 \end{aligned}$$

  See the lecture notes for more examples.