# Automated Reasoning for SE: Lab 3

November 16, 2005

## 1 Task 1

Clausify the following propositional logic formulae.

$$\begin{array}{l}
\neg\neg A \rightarrow \neg(B \vee C) \\
\neg\neg(\neg\neg A \rightarrow A) \wedge \neg(A \vee B) \\
((A \vee C) \rightarrow ((A \rightarrow D) \rightarrow ((C \rightarrow C) \rightarrow D)) \\
(A \rightarrow B) \leftrightarrow (A \wedge B) \\
\neg\neg A \rightarrow (A \wedge B)
\end{array}$$

## 2 Task 2

Use propositional resolution to derive contradiction (the empty clause) from the following clause set:

$$\{P \vee P, \neg P \vee \neg Q, Q \vee \neg A, A\}$$

Give each inference step.

## 3 Task 3

Causify the following first-order formulae:

$$\begin{array}{ll}
(a) & \forall x(P(x) \vee R(x) \rightarrow \exists y \forall w Q(y, w, x)) \\
(b) & \forall x \forall w(P(x) \vee R(x) \rightarrow \exists y Q(y, w))
\end{array}$$

## 4 Task 4

Use first-order resolution to derive the empty clause from the following clause sets. If necessary, clausify the formulae first.

- (a)

$$\begin{array}{ll}
(1) & \neg P(x, f(y)) \vee P(y, f(x)) \\
(2) & P(f(a), f(a)) \\
(3) & \neg P(z, f(f(z)))
\end{array}$$

- (b)

$$\begin{array}{ll} (1) & \neg blue(x) \vee \neg green(x) \\ (2) & green(A) \\ (3) & blue(C) \\ (4) & on(A, B) \\ (5) & on(B, C) \\ (6) & \neg on(x, y) \vee \neg green(x) \vee green(y) \end{array}$$

  Here $x, y$ are variables, and $A, B, C$ are constants.

- (c)

$$\begin{array}{ll} (1) & \forall x, y(Q(x, y) \rightarrow \forall z(R(z, y) \vee R(x, z))) \\ (2) & \forall u \exists v Q(u, v) \\ (3) & \neg \forall y \exists w R(y, w) \end{array}$$

- (d)

$$\begin{array}{ll} (1) & \forall x(\forall y(Child(y, x) \rightarrow Fly(y)) \wedge Dragon(x) \rightarrow Happy(x)) \\ (2) & \forall x(Green(x) \wedge Dragon(x) \rightarrow Fly(x)) \\ (3) & \forall x(\exists y(Parent(y, x) \wedge Green(y)) \rightarrow Green(x)) \\ (4) & \forall z \forall x(Child(x, z) \wedge Dragon(z) \rightarrow Dragon(x)) \\ (5) & \forall x \forall y(Child(y, x) \rightarrow Parent(x, y)) \\ (6) & \text{The negated conclusion:} \\ & \neg \forall x(Dragon(x) \rightarrow (Green(x) \rightarrow Fly(x))) \end{array}$$