Young Applied Analysts in the UK

Titles and Abstracts

University of Bath, 26-27 May 2016

Conditions for the Hölder regularity of local minimizers of a nonlinear elastic energy in two dimensions

Jonathan Bevan

We present conditions sufficient for the Hölder regularity of minimizers of an energy which can model isotropic materials, including rubber. By hypothesizing that deformations map circles to suitably star-shaped sets, and then by applying an adapted elliptic regularity theory, we can overcome some of the technical difficulties that have long been associated with proving regularity of elastic energy minimizers. A practical demonstration of the theory is given in the class of 'shear maps'.

The rigidity problem for symmetrization inequalities

Filippo Cagnetti
(joint work with Maria Colombo, Guido De Philippis, Francesco Maggi, Matteo Perugini, Dominik Stoger)

We will discuss several symmetrizations (Steiner, Ehrhard, and spherical symmetrization) that are known not to increase the perimeter. We will show how it is possible to characterize those sets whose perimeter remains unchanged under symmetrization. We will also characterize rigidity of equality cases. By rigidity, we mean the situation when those sets whose perimeter remains unchanged under symmetrization, are trivially obtained through a rigid motion of the (Steiner, Ehrhard or spherical) symmetral. We will achieve this through the introduction of a suitable measure-theoretic notion of connectedness, and through a fine analysis of the barycenter function for a special class of sets.

Diffusive Dominated Keller-Segel Model

José Antonio Carrillo de la Plata

We analyze under which conditions equilibration between two competing effects, repulsion modeled by nonlinear diffusion and attraction modeled by nonlocal interaction, occurs. This balance leads to continuous compactly supported radially decreasing equilibrium configurations for all masses. All stationary states with suitable regularity are shown to be radially symmetric by means of continuous Steiner symmetrization techniques. Calculus of variations tools allow us to show the existence of global minimizers among these equilibria. Finally, in the particular case of Newtonian interaction in two dimensions they lead to uniqueness of equilibria for any given mass up to translation and to the convergence of solutions of the associated nonlinear aggregation-diffusion equations towards this unique equilibrium profile up to translations as $t \to \infty$.

On gaps in the spectrum of the periodic Maxwell operator

Shane Cooper
(joint work with I. Kamotski and V. Smyshlyaev)

We discuss the propagation of electromagnetic waves in a photonic crystal fibre. We demonstrate the existence of gaps in the spectrum of the periodic Maxwell operator with mildly contrasting coefficients and finite period. The dependence on the location of these gaps with respect to the geometry of the media is studied.

Homogenisation for non-coercive functionals with random coefficients under anisotropic rescaling

Federica Dragoni
(joint work with Nicolas Dirr, Claudio Marchi and Paola Mannucci)

In the talk I Investigate the limit behaviour for a family of non coercive functionals with random coefficients. I will first describe a lower dimensional constrained variational problem and then I will use the Hörmander condition to recover the result for the unconstrained functional. The rescaling will be anisotropic to adapt to the new underlying degenerate geometrical structure (Carnot groups). Under a suitable stationary-ergodic assumption, the solutions of the stochastic microscopic models will converge to a function independent of the random variable.

As application one can derive a similar homogenisation result for Hamilton-Jacobi problems with random coefficients. In fact the variational problem can be interpret as the unique viscosity solution of some associated first-order PDEs.

Why optimal transport theory for repulsive costs?

Augusto Gerolin

The goal of this communication is to present a new class of optimal transport problems with repulsive cost functions motivated by Quantum Mechanics (computation of the ground state energy of an Electronic Schrödinger Equation). I want to highlight some issues and point out our progress regarding the existence of Monge-type solutions in this context.

Free boundary regularity in two phase variational problems
Aram Karakhanyan

I will give an overview of the regularity theory of two phase free boundary problems for both variational and viscosity solutions. When the governing PDE is linear, say it is the Laplace operator, then many powerful tools and results are available such as the Alt-Caffarelli-Friedman and Weiss monotonicity formulae. However for nonlinear PDEs the theory is widely open. Recently in a joint work with Serena Dipierro we made some advances in this directions and will discuss our result in the second part of the talk.

A Max-Cut Approximation Using A Graph Based MBO Scheme
Blaine Keetch

On a given unweighted graph $G = (V; E)$ a Max-Cut is a partition of nodes $V = V_1 \cup V_2$ such that the number of edges between the node sets V_1 and V_2 are maximized. This problem was proved to be NP-Complete by Richard Karp in 1972. Due to the complexity of the problem there are various types of approximation algorithms designed to obtain an accurate estimate of the Max-Cut.

In this talk I will discuss an alternate graph-based Ginzburg-Landau functional and how the minimization of this functional relates to a graph's Max-Cut; we attempt to minimize this functional by a graph-based MBO Scheme, where we use a graph heat equation with a twist, instead of diffusion being defined using a graph Laplacian we use an operator called the signless Laplacian. All of these concepts will be discussed briefly.

Global Jacobians and boundary vortices
Matthias Kurzke
(joint work with Radu Ignat)

In the theory of Ginzburg-Landau vortices, a natural quantity with good compactness properties is the Jacobian of a map. For vortices approaching the boundary, however, a natural stability property breaks down. The problem disappears when we study a natural distributional version, the global Jacobian, which can deal with situations involving interior and boundary vortices. As an application, I will discuss a thin-film limit of micromagnetics that involves boundary vortices.

The Well Order Reconstruction Solution for Nematic Liquid Crystals

Apala Majumdar

(joint work with Samo Kralj, Giacomo Canevari, Amy Spicer, Martin Robinson, Chong Luo and Radek Erban)

We analyse a new well order reconstruction solution in the Landau-de Gennes theory for nematic liquid crystals in terms of a saddle-type critical point of an associated scalar variational problem. This solution was reported numerically in three-dimensional square wells, by Kralj & Majumdar in 2014. We prove the existence of this well order reconstruction solution, study its qualitative properties and prove that the solution loses stability for large wells, bifurcating into the more familiar diagonal and rotated solutions for large wells.

A regularity result for the *p*-laplacian near uniform ellipticity

Carlo Mercuri

(joint work with B. Sciunzi and G. Riey)

We consider weak solutions to a class of Dirichlet boundary value problems involving the p-Laplace operator, and prove that the second weak derivatives are in L^q with q as large as it is desirable, provided p is sufficiently close to $p_0 = 2$. We show that this phenomenon is driven by the classical Calderón-Zygmund constant. As a byproduct of our analysis we show that $C^{1,\alpha}$-regularity improves up to $C^{1,1-}$, when p is close enough to 2.

Interfacial energies of systems of chiral molecules

Mariapia Palombaro

(joint work with A. Braides and A. Garroni)

I will discuss the derivation of a macroscopic model for the assembly of chiral molecules in two dimensions driven by maximization of the contact area. The macroscopic model is described by a parameter taking nine possible values corresponding to the possible minimal microscopic patterns and modulated phases of the chiral molecules. The overall behaviour is then described by means of an interaction energy of perimeter type between such phases. This energy is a crystalline perimeter energy, highlighting preferred directions for the interfaces between ensembles of molecules labelled by different values of the parameter.

Convergence of a particle method for diffusive gradient flows in one dimension

Francesco Patacchini

(joint work with J. A. Carrillo, Y. Huang, P. Sternberg and G. Wolansky)

We approximate diffusive gradient flows with finite numbers of particles in one dimension. As the quadratic Wasserstein energy is not defined for point-masses, we spread uniformly the mass of each particle in some interval around it. This "tessellation" gives a discrete energy defined on point-masses, which Gamma-converges in the Wasserstein topology to its continuum version as the number of particles increases. Using an abstract result for the convergence of gradient flows in metric spaces by S. Serfaty, we show the convergence of the resulting discrete gradient flow to the continuum one.

Stochastic homogenization of a model of plant biomechanics

Mariya Ptashnyk

In this talk we consider a microscopic model for biomechanics of a plant tissue with randomly distributed cells. Applying techniques of stochastic homogenization we derive macroscopic equations that allow us to analyse the impact of random microscopic structure on the macroscopic behaviour and macroscopic mechanical properties of plant tissues. The main challenge in the stochastic homogenization of models for plant biomechanics comes from the perforated structure of plant tissues and boundary conditions on surfaces of the microstructure.

Fractional Harmonic Maps

James Roberts

Fractional harmonic maps are an extension of the notion of harmonic mappings of Riemannian manifolds and an analogue of solutions of $(-\Delta)^s u = 0$ for $s \in (0, 1)$. They arise as critical points of energies which are intimately connected to the Dirichlet energies

$$\frac{1}{2}\int_{\mathbb{R}^m \times (0,\infty)} x_{m+1}^{1-2s} |\nabla v|^2 \mathrm{d}x$$

where the factor x_{m+1}^{1-2s} may degenerate or become singular along $\mathbb{R}^m \times \{0\}$ depending on s. Using this relationship it is possible to discern some properties of fractional harmonic maps, such as their regularity, by considering the corresponding properties of their extensions to a half-space. I will introduce the concept of fractional harmonic maps and illustrate their regularity properties. ?

Centroidal power diagrams: Numerical analysis and applications

Steven Roper

We present numerical analysis of a variant of Lloyd's algorithm for the computation of centroidal power diagrams. A power diagram (or Laguerre-Voronoi diagram) is a type of weighted Voronoi diagram that arises in, for example, consideration of certain problems in pattern formation in materials science.

On the Emergence of Surface Energies in a Microscopic, Two-Dimensional Two-Well Problem

Angkana Rüland

(joint work with G. Kitavtsev and S. Luckhaus)

In this talk I present and analyze a $SO(2)$-invariant, two-well Hamiltonian on a 2D atomic lattice for low energy states with surface energy scaling. The two wells of the Hamiltonian consist of the $SO(2)$ orbits of two rank-one connected matrices. Seeking to obtain an understanding of the origin of surface energies, I first focus on "atomic chains", i.e. on special "$(1+\epsilon)$-dimensional" states . For these I explain the formation of the twinning and boundary layers. If time permits I will also discuss a full two-dimensional variant of the problem.

Properties of minimizers of the average-distance problem

Dejan Slepčev

The average distance problem asks to find a good way to approximate a high-dimensional object, represented as a measure, by a one-dimensional object. We will discuss two variants of the problem: one where the one-dimensional object is a measure with connected one-dimensional support and one where it is an embedded curve. We will discuss examples that show that even if the data measure is smooth the nonlocality of the functional can cause the minimizer to have corners. Nevertheless the curvature of the minimizer can be considered as a measure. We will discuss a priori estimates on the total curvature and ways to obtain information on topological complexity of the minimizers.

The Derivation of the Linear Boltzmann Equation from a Rayleigh Gas Particle Model

George Stone

We derive the linear Boltzmann equation from a Rayleigh gas particle model in the Boltzmann-Grad limit for arbitrary long times under moderate assumptions on the moments of the initial distributions. Convergence is shown by studying Kolmogorov equations for associated probability measures on collision histories.

Tba

Dimitrios Tsagkarogiannis

Orientational Ordering and Rigidity in 2 Dimensions

Luke Williams

A long standing problem in statistical physics is attempting to show that matter can undergo crystallisation. Recently there has been some progress showing that a simple atomistic model possesses an ordered phase using a rigidity estimate, but the ordering cannot be controlled uniformly and no large volume limits can be taken. My talk will discuss a simple improvement that can be made to make the estimate uniform, at the cost of excluding some possible defects that inhibit crystallisation.