MORITA EQUIVALENCE OF SEMIGROUPS WITH LOCAL UNITS

M. V. LAWSON

In kindergarten, we learn that structures in mathematics should ideally be classified up to isomorphism. Later we discover that in most cases this ideal is impossible to achieve and we have to be less ambitious. Topology points the way: spaces are classified up to homotopy equivalence rather than homeomorphism but nevertheless deep theorems can be proved. Analogously, algebraic structures have to be classified by an equivalence that is weaker than isomorphism but with enough strength to be meaningful.

In the theory of unital rings, Morita theory provides the correct classifying notion [9]: two unital rings are Morita equivalent if their categories of left modules are equivalent. This categorical definition can be characterized in purely algebraic terms: unital rings R and S are Morita equivalent if and only if R is isomorphic to a full corner of the ring of all matrices over S of some fixed finite size. This result contains, as a special case, the classical Artin-Weddurburn theorem.

The Morita theory of monoids was introduced independently by Banaschewski [2] and Knauer [6]. Monoids S and T are Morita equivalent if their categories of left 'modules' are equivalent. This categorical definition can also be characterized in purely algebraic terms: monoids S and T are Morita equivalent if and only if there is an idempotent e in T, where $T = TeT$, such that S is isomorphic to eTe. This is similar to the case of unital rings but without the matrices.

The generalization of Morita theory to structures without identities is difficult. In this talk, we are interested in the generalization of the Morita theory of monoids to more general classes of semigroups. The first, and decisive, step in carrying out this generalization was due to Talwar [22, 23, 24] who was motivated by the work of Abrams [1] in the extension of Morita theory from unital to more general kinds of rings. Talwar defined a Morita theory for semigroups with local units, where a semigroup S is said to have *local units* if for each $s \in S$ there exist idempotents e and f such that $es = s = sf$. Both monoids and regular semigroups have local units.

Despite this first success, only a few papers were subsequently written developing Talwar's ideas [3, 18, 19], perhaps because Talwar did not provide a corresponding algebraic characterization of his notion of Morita equivalence.

Recently, however, there have been new developments. Steinberg introduced a 'strong' Morita theory for inverse semigroups [21], which turns out to be the same as the usual Morita theory of inverse semigroups, although in a form better adapted to inverse semigroups [5]; and Laan and Márki [8] have been exploring Morita theory for various classes of semigroups including generalizations of semigroups with local units.

In this talk, I shall describe Talwar's theory and then show that it can be characterized algebraically in terms of the notion of an enlargement [10]. As an application, I shall show that the theory of the local structure of regular semigroups developed by McAlister [13, 14, 15, 16, 17] can be viewed as a contribution to the Morita theory of regular semigroups, and as a direct generalization of the pioneering paper of Rees [20]. Specifically, completely simple semigroups are the semigroups Morita equivalent to groups, and the locally inverse semigroups are those Morita equivalent to inverse semigroups.

The talk will be aimed at a general algebraic audience.

References

[1] G. D. Abrams, Morita equivalence for rings with local units, *Comm. Algebra* **11** (1983), 801–837.

[2] B. Banaschewski, Functors into categories of M-sets, *Abh. Math. Sem. Univ. Hamburg* **38** (1972), 49–64.

[3] Y. Q. Chen, K. P. Shum, Morita equivalence for factorisable semigroups, *Acta Math. Sin.* **17** (2001), 437–454.

[4] B. Elkins, J. A. Zilber, Categories of actions and Morita equivalence, *Rocky Mountain J. Math.* **6** (1976), 199–225.

[5] J. Funk, M. V. Lawson, B. Steinberg, Characterizations of Morita equivalence of inverse semigroups, Preprint, 2010.

[6] U. Knauer, Projectivity of acts and Morita equivalence of monoids, *Semigroup Forum* **3** (1972), 359–370.

[7] V. Laan, More on strong Morita equivalence of semigroups with local units,

[8] V. Laan, L. Márki, On strong Morita equivalence of semigroups, in preparation.

[9] T. Y. Lam, *Lectures on rings and modules*, Springer-Verlag, New York, 1999.

[10] M. V. Lawson, Enlargements of regular semigroups, *Proc. Edinb. Math. Soc.* **39** (1996), 425–460.

[11] M. V. Lawson, Atlases, inverse semigroups and Morita equivalence, Preprint 2010.

[12] M. V. Lawson, L. Márki, Enlargements and coverings by Rees matrix semigroups, *Monatsh. Math.* **129** (2000), 191–195.

[13] D. B. McAlister, Regular Rees matrix semigroups and regular Dubreil-Jacotin semigroups, *J. Aust. Math. Soc. (Series A)* **31** (1981), 325–336.

[14] D. B. McAlister, Rees matrix covers for locally inverse semigroups, *Trans Amer. Math. Soc.* **277** (1983), 727–738.

[15] D. B. McAlister, Rees matrix covers for regular semigroups, *J. Algebra* **89** (1984), 264–279.

[16] D. B. McAlister, Rees matrix covers for regular semigroups, in *Proceedings of 1984 Marquette Conference on Semigroups* (editors Byleen, Jones and Pastijn), Marquette University, Milwaukee, 1985, 131–141.

[17] D. B. McAlister, Quasi-ideal embeddings and Rees matrix covers for regular semigroups, *J. Algebra* **152** (1992), 166–183.

[18] V. V. Neklyudova, Polygons under semigroups with a system of local units, *Fundamental and Applied Mathematics* **3** (1997), 879–902 (in Russian).

[19] V. V. Neklyudova, Morita equivalence of semigroups with a system of local units, *Fundamental and Applied Mathematics* **5** (1999), 539–555 (in Russian).

[20] D. Rees, On semi-groups, *Proceedings of the Cambridge Philosophical Society* **36** (1940), 387–400.

[21] B. Steinberg, Strong Morita equivalence of inverse semigroups, To appear in *Houston J. Math.*

[22] S. Talwar, Morita equivalence for semigroups, *J. Aust. Math. Soc. (Series A)* **59** (1995), 81–111.

[23] S. Talwar, Strong morita equivalence and a generalisation of the Rees theorem, *J. Algebra* **181** (1996), 371–394.

[24] S. Talwar, Strong morita equivalence and the synthesis theorem, *Internat. J. Algebra Comput.* **6** (1996), 123–141.

Department of Mathematics, and the, Maxwell Institute for Mathematical Sciences, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS, Scotland
E-mail address: markl@ma.hw.ac.uk