Lecture 1

Algebra — manipulating symbols

calculations too hard — draw pictures

Pictures too hard - use symbols

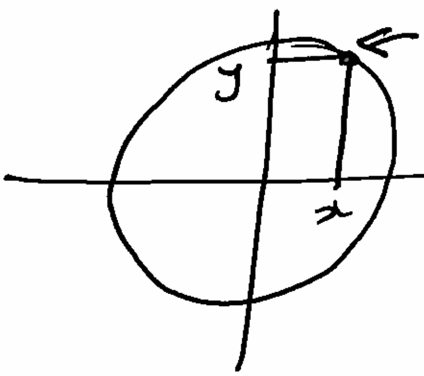
Geometry — drawing pictures

Algebraic question

Example 1

Solve $x^2+y^2=1$, That is find all real numbers x and y that satisfy this equation.

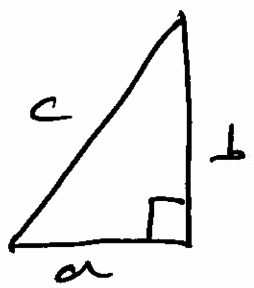
Descartes & Fermat introduced a coordinate system (called Cartesian after Descartes) so that the solutions to the above equation described a circle in the plane.



(x,y) & $x^2+y^2=1$

<u>Example 2</u> Algebraic question.

Find all positive whole numbers a, b, c such that $a^2 + b^2 = c^2$. Such a triple of numbers is called a <u>Pythagorean triple</u>



They describe right-angled triangles with sides that are whole numbers.

<u>Connection with circles</u>

Given Pythagorean triple (a, b, c) so that $a^2 + b^2 = c^2$, we know $c \neq 0$ and so $c^2 \neq 0$, so we can divide through by c^2 to get

$$\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$$

Thus $\left(\frac{a}{c}, \frac{b}{c}\right)$ lies on the unit circle <u>and</u> has <u>rational coordinates</u>.

Let $\left(\frac{a}{b}, \frac{c}{d}\right)$ be a point on the unit circle with rational coordinates.

Then $\frac{a^2}{b^2} + \frac{c^2}{d^2} = 1$.

It follows that $a^2d^2 + c^2b^2 = b^2d^2$

Thus (ad, cb, bd) is a Pythagorean triple. It follows that Pythagorean triples are somehow (and there is more work to be done) connected with rational points on the unit circle.

more work needs to be done to make this precise.

— We can solve $a^2 + b^2 = c^2$ in whole numbers, but what about $a^n + b^n = c^n$ (where $n \geq 3$). This is the subject of Fermat's Last Theorem.

One theme of this course is different kinds of numbers and how to calculate with them.

- <u>natural numbers</u> $0, 1, 2, 3, \ldots$
- <u>integers</u> $\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
- <u>rationals</u> all numbers that can be written $\frac{p}{q}$ where $q \neq 0$, p, q integers

<u>Real numbers</u> all numbers that can be written in the form $a \cdot a_1 a_2 a_3 \ldots$ i.e. as decimals with possibly infinitely many digits after the decimal point.