Lecture 14

Example (of calculating square roots of complex numbers) Calculate the square roots of $3+4i$

Let $(x+iy)^2 = 3+4i$ $(*)$

Thus $(x^2-y^2) + 2xyi = 3+4i$

Equate real and imaginary parts

① $x^2 - y^2 = 3$

② $2xy = 4$

To get the third equation take moduli of $(*)$

$$|x+iy|^2 = |3+4i|$$

③ $x^2+y^2 = \sqrt{9+16} = \sqrt{25} = 5$

① + ③

$$2x^2 = 8$$

so $x^2 = 4$

$x = \cancel{4}$ or -2

when $x = 2$, $y = 1$
when $x = -2$, $y = -1$ } by ②

Thus the two square roots are

$2+i$ and $-2-i$

Check $(2+i)^2 = 3+4i$

If $z \neq 0$, its square roots are always

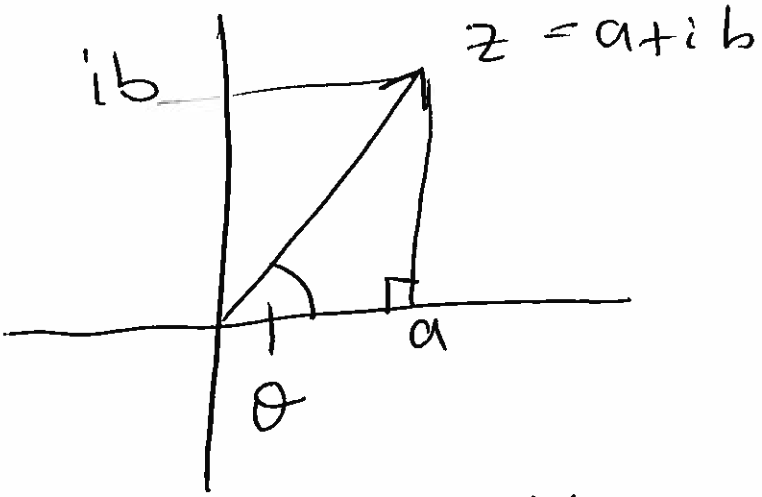
of the form $u, -u$.

The proof of the following is a result of the existence of square roots and the usual formula for the roots of a quadratic (or completing the square)

Theorem

Every quadratic equation with complex coefficients has exactly two roots (counting multiplicities).

Complex no. geometry



If $z \neq 0$, we can write

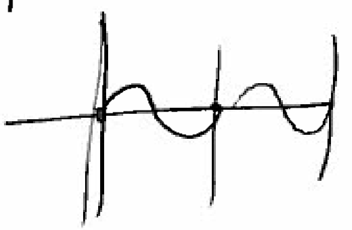
$$\boxed{z = |z| (\cos\theta + i\sin\theta)}$$

called polar form

The angle θ is called an <u>argument</u> of z.

$$\arg z = \{ \theta + 2\pi k : k \in \mathbb{Z} \}$$

ie θ is not unique because sin and cos are periodic functions with period 2π.



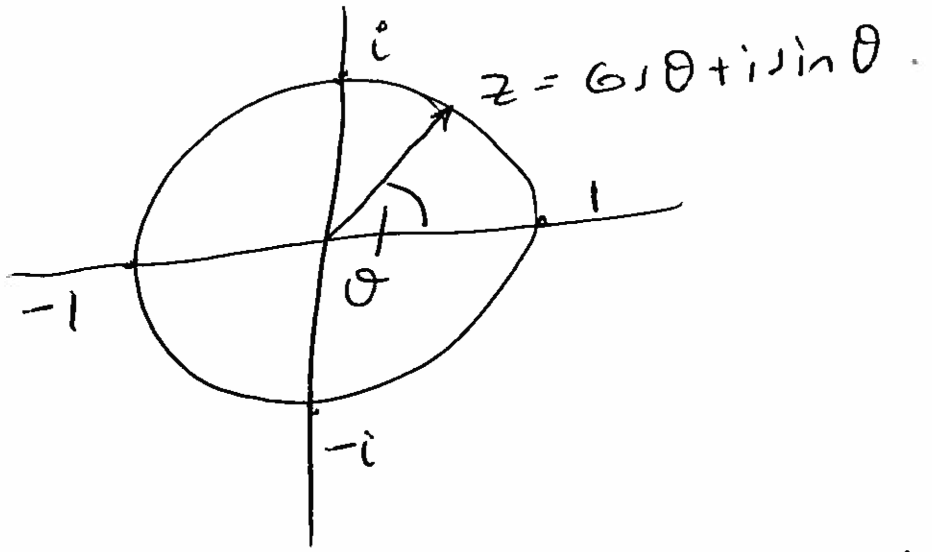
Let's multiply together two complex numbers in polar form.

$$u = r(\cos\theta + i\sin\theta)$$
$$v = s(\cos\phi + i\sin\phi)$$

$$uv = rs(\cos(\theta+\phi) + i\sin(\theta+\phi))$$

using the usual trig addition formulae.

<u>Example</u> The complex numbers of the form $\cos\theta + i\sin\theta$ are precisely those with unit modulus.



If you multiply any two numbers of unit modulus you get another one.

$$(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$$

more generally

<u>De Moivre's theorem</u> $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

De Moiver's theorem can be used to obtain trig identities easily.

<u>Example</u> $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

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$\cos^2\theta + 2\cos\theta\sin\theta\, i + i^2\sin^2\theta$

equate real and imaginary part,

$$\boxed{\begin{aligned} \cos 2\theta &= \cos^2\theta - \sin^2\theta \\ \sin 2\theta &= 2\cos\theta\sin\theta . \end{aligned}}$$