Lecture 5

Sets and Counting

applications to probability theory

Define the empty set $\{\} = \emptyset$

nothing there

Observe that $\emptyset \neq \{\emptyset\}$

no elements

one element

If X is a set, denote by $|X|$ the cardinality of X. That is, the number of elements in X.

Sets are defined by saying what properties the elements of the set must possess.

$$\{x : P(x)\}$$

the set

of all x

such that

x satisfies the property P

Examples

(1) $A = \{x : x \in \mathbb{N} \text{ and } x \text{ is even}\}$
$= \{0, 2, 4, 6, \ldots\}$

(2) $B = \{x : x \in \mathbb{N} \text{ and } x^2 = 1\}$
$= \{1\}$

(3) $C = \{x : x \in \mathbb{Z} \text{ and } x^2 = 1\}$
$= \{-1, 1\}$

(4) $D = \{x : x \in \mathbb{R} \text{ and } x^2 = -1\}$
$= \emptyset$

X a set.
Define $P(X)$, called the <u>power set</u> of X, to be the set of all subsets of X.

<u>Example</u> Calculate $P(\{a,b,c\})$.

$$P(\{a,b,c\}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$$

Some interesting patterns arise.

This generalizes

subsets	number
0 elements	1
1 element	3
2 elements	3
3 elements	1

1 3 3 1 This is a row of <u>Pascal's $\triangle$</u>.

Observe that $1+3+3+1 = 8 = 2^3$

Let X be a set with n elements. A subset $A \subseteq X$ with k elements is called a <u>k-subset</u> $(0 \le k \le n)$.

The number of k-subsets of an n-element set is denoted by

Calculator $^{n}C_{k}$ read 'n choose k'

Maths notation $\binom{n}{k}$ called a binomial coefficient

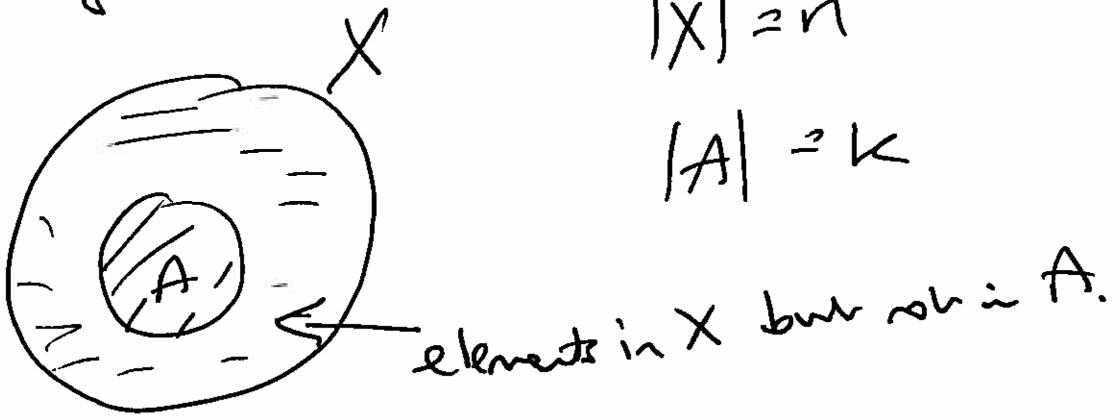
$$\boxed{^{n}C_{k} = \frac{n!}{k!\,(n-k)!}}$$

← This is proved in my books, but I will assume it here.

$[n! = n(n-1)\ldots 1\ ;\ 0! \overset{\text{def}}{=} 1]$

Example How many committees (!) can be formed from 170 students consisting of 4 members. Solution: $\binom{170}{4}$.

Example $\binom{n}{k} = \binom{n}{n-k}$. This can be proved by algebra but there is a nice way of seeing why this is true.



$|X| = n$

$|A| = k$

every time you choose a subset with k elements you are rejecting a subset with $n-k$ elements.

$\{a,b\}$ does not record order or repetitions. Thus $\{a,b\} = \{b,a\} = \{a,b,b\}$.

We shall now introduce some notations that will be sensitive to both order and repetitions.