Lecture 7

There are n positions and you have to choose i to fill with xs

There is an important consequence of our proof that $|P(X)| = 2^{|X|}$.

> The number of strings over $\{x, y\}$ of length n with exactly i xs is $\binom{n}{i}$

Example All strings of xs and ys of length 4 with 2 xs are

$\{xxyy, xyyx, xyxy, yxyx, yyxx, yxxy\}$

$\binom{4}{2} = \frac{4!}{2!2!} = \underline{\underline{6}}$

We use this result to prove the binomial theorem.

A nice application of Counting is the binomial theorem.

Binomial theorem

$$(x+y)^n = \sum_{i=0}^{n} \binom{n}{i} x^i y^{n-i}$$

where do those numbers come from

We distributivity ~~and~~ but do not apply commutativity of multiplication. and commutativity of addition.

Example

$$(x+y)^3 = xxx + (xxy + xyx + yxx) + (xyy + yxy + yyx) + yyy$$

- all strings in $xxy + xyx + yxx$ equal x^2y when commutativity of multiplication is used. Why $\underline{\underline{3}}$ terms $=$ # strings of xs and ys with two xs.

$$= \binom{3}{2}$$

- Similar argument ~~in~~ for $xyy + yxy + yyx$.

Example We expand $(2a-b)^{20}$ and calculate the coefficient of a^3.

$2a-b = 2a+(-b)$.

Thus
$$(2a+(-b))^{20} = \sum_{i=0}^{20}\binom{20}{i}(2a)^i(-b)^{20-i}$$
$$= \sum_{i=0}^{20}\binom{20}{i}2^i(-1)^{20-i}a^i b^{20-i}$$

The coefficient of a^3 is $\binom{20}{3}2^3(-1)^{17}$
$$= -\binom{20}{3}2^3 = -\frac{20!}{3!\,17!}\,8 = -\frac{20\cdot 19\cdot \cancel{18}^{\,6}\cdot \cancel{8}^{\,4}}{\cancel{3}\cdot\cancel{2}} = -9120$$

K-permutations

Given a set with n elements. A k-tuple of such elements without repeats is called a k-permutation

Example Let $X = \{a, b, c\}$. The set of 2-permutations of X is $\{ab, ac, ba, bc, ca, cb\}$.

The number of k-permutations of an n-element set is

$$^nP_k = \frac{n!}{(n-k)!}$$

If $k = n$, we simply talk of a permutation of n objects.

Example Permutations of the set $\{a, b, c\}$ is

$\{abc, acb, bac, bca, cab, cba\}$

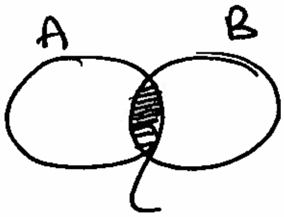
Summary of counting

	order	no order
repetitions	<u>K-tuples</u> k^n	not discussed
no repetitions	<u>permutations</u> ${}^nP_k$	<u>Combinations</u> ${}^nC_k$

$$\boxed{{}^nP_k = k!\;{}^nC_k}$$

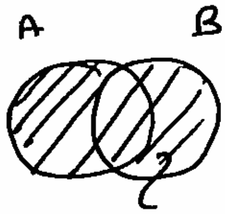
Boolean operations on sets

We shall use Venn diagrams.



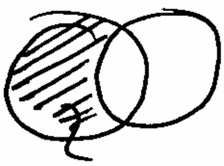
$A \cap B = \{x : x \in A \text{ and } x \in B\}$

$A \cap B$ = Intersection of A and B



$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$A \cup B$ = union of A and B



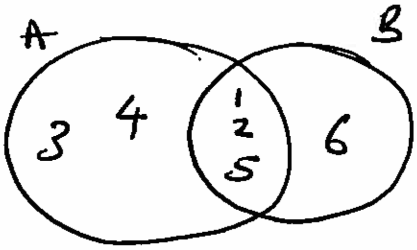
$A \setminus B = \{x : x \in A,\ x \notin B\}$

$A \setminus B$

set difference

Example

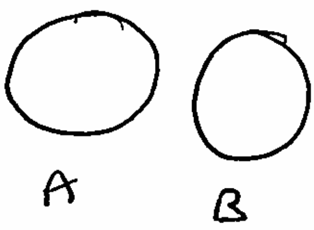
$A = \{1, 2, 3, 4, 5\},\ B = \{1, 2, 5, 6\}$



$A \cap B = \{1, 2, 5\}$
$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \setminus B = \{3, 4\}$
$B \setminus A = \{6\}$

Sets A and B are called disjoint if

$A \cap B = \emptyset$



If A and B are disjoint then
$|A \cup B| = |A| + |B|$