# Lecture 8

## Properties of Boolean operations

$$(B1) \quad A \cap (B \cap C) = (A \cap B) \cap C$$

associativity

$$(B2) \quad A \cap B = B \cap A$$

Commutativity

$$(B3) \quad A \cap \emptyset = \emptyset = \emptyset \cap A$$

$$(B4) \quad A \cup (B \cup C) = (A \cup B) \cup C$$

associativity

$$(B5) \quad A \cup B = B \cup A$$

Commutativity

$$(B6) \quad A \cup \emptyset = A = \emptyset \cup A$$

identity for $\cup$

$$(B7) \quad A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$(B8) \quad A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

distributivity

(B9) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

(B10) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

de Morgan's laws

(B11) $A \cap A = A$

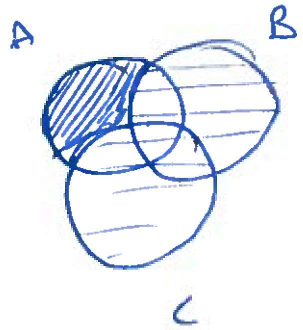
(B12) $A \cup A = A$

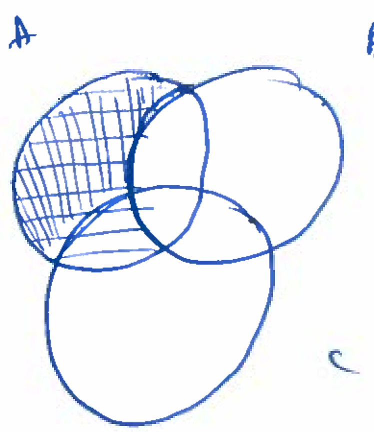
Idempotence.

These results can be illustrated by means of Venn diagrams.

Example $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$A \setminus (B \cup C)$

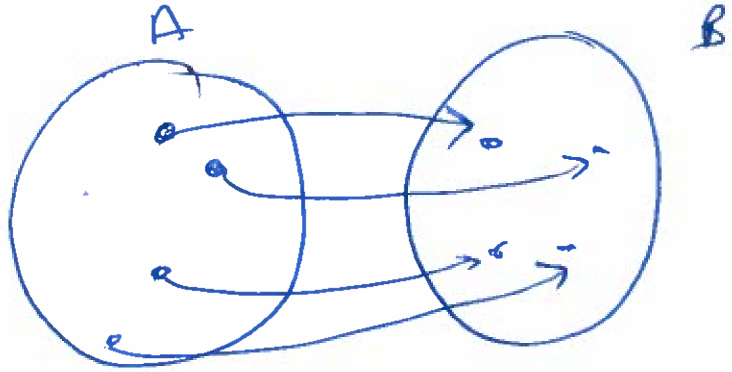


Remarks: Because of associativity, we can write $A_1 \cup \ldots \cup A_n$ unambiguously without brackets. Similarly, $A_1 \cap \ldots \cap A_n$ is unambiguous.

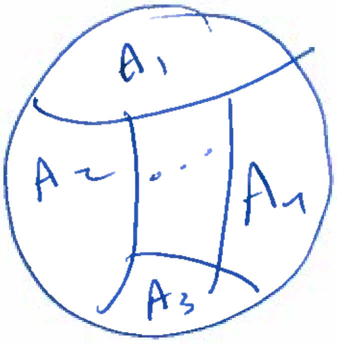
## Counting principles

(1) To count a complicated set $A$ find a simpler set $B$ such that there is a bijection between $A$ and $B$.




2) $|A_1 \times \ldots \times A_n| = |A_1| \ldots |A_n|$.
Product counting principle

3) Partition counting principle.




Let $A$ be a set.

$P = \{A_1, \ldots, A_n\}$ a set of subsets of $A$ is called a <u>partition</u> if

(1) $A = A_1 \cup \ldots \cup A_n$

(2) $A_i \neq \emptyset \quad 1 \leq i \leq n$.

(3) $A_i \cap A_j = \emptyset$ if $i \neq j$

$$|A| = |A_1| + \ldots + |A_n|.$$

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