Autumn 2015 Algebra A F17CC

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Aims The goal is to provide a bridge between school and university mathematics. There is some overlap with A-level and Advanced Highers, though I assume you have neither. Although there will be elements of revision for many students, I shall introduce you to the notion of proof and why it is needed.

Quizzes There will be five quizzes in lectures. They will not count towards your final grade but they will give you a chance to see how you are getting along. I will go over the solutions to each quiz in lectures. All quiz papers will be marked during the lecture and then you will hand them in. This will give me a chance to see how you are doing and where you are having problems.

Exam There will be one final 2 hour exam paper worth 100%. There are also exam resits in the summer for those who need them. I set the exams based on the course I have taught and I also mark all exams. Exam questions have the same format as the exercise questions at the end of each section of my book. *There are no special types of exam questions.* Towards the end of the semester, I shall provide you with a copy of last year's exam paper and solutions. It is the policy of the Mathematics Department not to provide more than one past exam paper. We want you to learn and understand mathematics not play games trying to pass exams.

Material A *free* copy of my book, in pdf-format, will be posted on VISION. This covers much more than is in this course, but I will tell you what you have to study in the **Study Guide**. The *exercises* are at the end of each section of the book. Again, I will tell you what exercises you should attempt. *Solutions to exercises* will be provided in tutorials and nowhere else. The *website* for the course can be accessed via VISION or you can find it directly via my **homepage** (click **Teaching** then **Algebra A**).

Missing lectures etc If you miss a lecture it is your responsibility to catch up not the lecturer's. On the other hand, if you have to miss a lecture for any reason you do not need to tell me. But, if you miss several lectures for serious reasons, you are expected to contact your mentor and explain the situation to them. Maybe they can help.

Tutes start in week 2.

Revision I hope to give you about two weeks between the last new material and the exam where you can revise. I will not run special revision classes but I will provide you with surgery hours during those two weeks when you can ask

me individual questions.

Outline syllabus

Sets and counting Set theory is a simple language needed throughout mathematics. We shall use it also to provide a setting for some counting results. *Where used: probability theory.*

Number theory We shall describe the different kinds of numbers used in mathematics together with some results about them. *Where used: cryptography.*

Complex numbers and polynomials An introduction to complex numbers and their arithmetic with an application to the study of polynomials. *Where used: throughout mathematics particularly in calculus.*

Matrices An introduction to matrices with applications to solving systems of linear equations. *Where used: throughout mathematics. This is probably the single most purely useful part of this course.*

Vectors An introduction to 3D vectors from scratch and their applications in geometry. *Where used: mechanics and vector calculus.*

Books for further reading and practice

If you need extra practice above and beyond the exercises to be found in the book then at university it is your responsibility to carry this out in your own time. The books below are useful sources of further problems.

R. Hammack, *Book of proof*, VCU Mathematics Textbook Series, 2009. This book can be downloaded for free from

`http://www.people.vcu.edu/~rhammack/BookOfProof/index.html`

or click the link on the website for this course. This is an excellent reference for material dealing with sets and counting, in addition for more information about proofs in mathematics.

A. Hirst, D. Singerman, *Basic algebra and geometry*, Pearson Education, 2001. This is a textbook with the same intentions as this course and would be useful as a source of different approaches and further examples. It covers much the same ground as my course, though oddly does not cover the solution of linear equations by Gaussian elimination.

S. Lipschutz, M. Lipson, *Discrete mathematics*, second edition and onwards, Schaum Outline Series. As with all the Schaum books, this is an excellent place to look for worked examples and further exercises. Chapters 1, 5 and 11 seem

the most relevant to this course.

S. Lipschutz and M. Lipson, *Linear Algebra*, third edition and onwards, Schaum Outline Series. Chapters 1, 2, 3 and 8 are covered in this course.

J. Olive, *Maths: a student's survival guide*, second edition, CUP, 2006. This book is primarily designed for science students but is, in fact, a very useful source for a lot of A-level/(Advanced) Highers standard mathematics that you may have forgotten from school or never encountered. In addition, it contains material that I will cover from scratch in this course in Chapters 1, 2, 3, 6, 7, 10, 11.

Information on the Web There's oodles of the stuff. Just use your favourite search-engine, such as Google, and type in key words that deal with the topic you are interested in. If you find anything you particularly like, let me know and I shall post the link on the course website. You can also use this method to find the mathematics department home-pages, my home-page or the university's homepage.

Library There are many maths books suitable for this course in the library. It is well-worth browsing and learning where everything is.

Learning outcomes

I. Sets and counting

- Manipulate sets and their elements.
- Answer simple counting questions involving permutations and combinations.
- Apply the Binomial theorem.

II. Number theory

- Convert between base 10 and another number base.
- Calculate greatest common divisors using Euclid's algorithm.
- Solve the linear Diophantine equation $ax + by = c$.
- Calculate least common multiples.
- Determine whether a number is prime or not.
- Find prime factorizations of numbers.
- Convert ultimately periodic decimals into rationals.

III. Complex numbers and polynomials

- Add, subtract, multiply and divide complex numbers.
- Find square roots of complex numbers.
- Solve quadratics by completing the square.
- Represent complex numbers in the complex plane.
- Understand the geometric interpretation of multiplication of complex numbers.
- Find nth roots.
- Use De Moivre's theorem to find expressions for $\sin^n \theta$ and $\cos^n \theta$.
- Understand the difference between trigonometric solutions and radical solutions.
- Find rational roots of polynomials with integer coefficients.
- Factorize real and complex polynomials appropriately.

IV. Matrices

- Add, subtract, and multiply two matrices, and multiply a matrix by a scalar; be able to carry out sequences of such operations to obtain a single matrix as a result.
- Solve linear equations using Gaussian elimination.
- Compute determinants by first row expansion.
- Compute matrix inverses using the adjugate method.
- Calculate the characteristic polynomial of a matrix and understand the statement of the Cayley-Hamilton theorem. Compute eigenvalues of matrices.

V. Vectors

- Compute with vectors using inner products, vector products, and scalar triple products.
- Find the equation of the unique line determined by two points or a point and a vector in space.
- Find the equation of the unique plane determined by three points or by a point and a normal.
- Calculate intersections of lines or planes.

- Find volumes of parallelepipeds using scalar triple products.

Conceptual aspects of mathematics

- Understand the need for proofs in mathematics.
- Understand key algebraic terms such as: associativity, commutativity, distributivity, identity, inverse.
- Understand the statements and rationale for the key theorems discussed in the course.