Exercises 2

(1) Prove the following four results.

Result 1: If $a_n > 0$ then $a_n x^n + \ldots + a_1 x + a_0 = O(x^n)$.

Result 2: If $0 \leq m < n$ then $x^m < x^n$ for all $x > 1$. In particular, $x^m = O(x^n)$.

Result 3: If $a > 1$ then $x^m = O(a^x)$ for all m.

Result 4: $\log_2(x) < x^\alpha$ for all $\alpha > 0$ for x large enough. In particular, $\log_2(x) = O(x^\alpha)$.

(2) Place the following functions in their correct order with respect to $f = O(g)$ i.e. fastest before slowest. You should justify your answers. You will find Question 1 helpful.

- n^2
- $\log n$
- $\log \log n$
- $(\log n)^2$
- $\sqrt{\log n}$
- $n!$
- n^n
- $n^2 \log n$
- $n \log n \log \log n$
- $n^{\frac{3}{2}}$
- 2^n
- 2^{n^2}
- 3^n
- $\log(n^2 + 1)$

(3) The goal of this question is to give you a sense of the difference between polynomial-time algorithms and exponential-time algorithms.

(a) A program has time complexity n^5. On a current computer the largest problem which can be solved in 1 hour is $n = N$. What is the size of the largest problem which can be solved in 1 hour by a machine which is 10,000 times faster?

(b) A program has time complexity 2^n. On a current computer the largest problem which can be solved in 1 hour is $n = M$. What is the size of the largest problem which can be solved in 1 hour by a machine which is 10,000 times faster?

(4) Estimate the time complexity of multiplying two $n \times n$ matrices together. Focus on the number of multiplications needed.

(5) Estimate the time complexity of searching for someone's telephone number in a telephone book.

(6) Use the algorithm from the lectures to determine whether the number 997 is prime or not.
(7) Find a proper factorization of the number $9,699,691$.