Exercises 4

(1) Show that 160 is invertible modulo 841 and calculate its inverse.
(2) Calculate 460^{179} modulo 2047.
(3) Find the unique solution to the following equations modulo 2640: $x \equiv 2 \pmod 3$, $x \equiv 3 \pmod 5$, $x \equiv 4 \pmod{11}$, $x \equiv 5 \pmod{16}$.
(4) Calculate $\phi(60)$.
(5) The number $n = pq$ where p and q are distint primes. Prove that if n and $\phi(n)$ are known then p and q are the roots of a quadratic.
(6) Determine which of the groups $\mathbb{U}_{10}$ and $\mathbb{U}_{12}$ is cyclic.
(7) Determine the orders of the elements in $\mathbb{U}_{16}$.
(8) Prove that the invertible elements of a monoid do indeed form a group.
(9) Prove that if two monoids are isomorphic then their groups of units are isomorphic.
(10) Prove that the relation $a \equiv b \pmod n$ really is an equivalence relation, and that the set of equivalence classes really does form a monoid under multiplication.