Exercises 11

(1) (a) Find the parametric and the non-parametric equations of the line through the two points with position vectors $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$.
(b) Find the parametric and the non-parametric equations of the plane containing the three points with position vectors $\mathbf{i}+3\mathbf{k}$, $\mathbf{i}+2\mathbf{j}-\mathbf{k}$ and $3\mathbf{i}-\mathbf{j}-2\mathbf{k}$.
(c) The plane $px+qy+rz=s$ contains the point with position vector $\mathbf{a}=2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ and is orthogonal to the vector $\mathbf{b}=\mathbf{i}+\mathbf{j}-\mathbf{k}$. Find $p, q, r$and s.

(2) *Intersection of two lines.*
(a) Find the vector equation of the line L_1 through the point with position vector $4\mathbf{i}+5\mathbf{j}+\mathbf{k}$ and parallel to the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$.
(b) Find the vector equation of the line L_2 through the point with position vector $5\mathbf{i}-4\mathbf{j}$ and parallel to the vector $2\mathbf{i}-3\mathbf{j}+\mathbf{k}$.
(c) Determine whether the two lines L_1 and L_2 intersect and if they do find the position vector of the point of intersection.

(3) *Intersection of two planes.* Find the vector equation of the line of intersection of the planes

$$x-y-2z=3 \text{ and } 2x+3y+4z=-2.$$

(4) *The distance of a point from a line* is defined to be the length of the perpendicular from the point to the line. Let the line in question have parametric equation $\mathbf{r}=\mathbf{p}+\lambda\mathbf{d}$ and let the position vector of the point be $\mathbf{q}$. Show that the distance of the point from the line is

$$\frac{\|\mathbf{d}\times(\mathbf{q}-\mathbf{p})\|}{\|\mathbf{d}\|}.$$

(5) *The distance of a point from a plane* is defined to be the length of the perpendicular from the point to the plane. Let the position vector of the point be $\mathbf{q}$ and the equation of the plane be $(\mathbf{r}-\mathbf{p})\cdot\mathbf{n}=0$. Show that the distance of the point from the plane is

$$\frac{|(\mathbf{q}-\mathbf{p})\cdot\mathbf{n}|}{\|\mathbf{n}\|}.$$

(6) *Spheres.* Let $\mathbf{c}$ be the position vector of the centre of a sphere with radius R. Let an arbitrary point on the sphere have position vector $\mathbf{r}$. Why is $\|\mathbf{r}-\mathbf{c}\|=R$? Squaring both sides we get $(\mathbf{r}-\mathbf{c})\cdot(\mathbf{r}-\mathbf{c})=R^2$. If $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ and $\mathbf{c}=c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$, deduce that the *equation of the sphere* with centre $c_1\mathbf{i}+c_2\mathbf{j}+c_3\mathbf{k}$ and radius R is $(x-c_1)^2+(y-c_2)^2+(z-c_3)^2=R^2$.
(a) Find the equation of the sphere with centre $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and radius 2.
(b) Find the centre and radius of the sphere with equation

$$x^2+y^2+z^2-2x-4y-6z-2=0.$$