
Exercises 8

(1) Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 4 \\ -1 & 1 \\ 0 & 3 \end{pmatrix}$.

(a) Calculate $-3B$.

(b) Calculate $A + B$.

(c) Calculate $A - B$.

(2) Let $A = \begin{pmatrix} 0 & 4 & 2 \\ -1 & 1 & 3 \\ 2 & 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -4 \\ 3 & 2 & 0 \end{pmatrix}$.

(a) Calculate AB .

(b) Calculate BA .

(3) Let $A = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 3 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 1 & 1 \end{pmatrix}$.

(a) Calculate BA .

(b) Calculate AA .

(c) Calculate CB .

(d) Calculate AC .

(e) Calculate BC .

(f) Calculate $C^T A$.

(4) Calculate

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}.$$

(5) Let $A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 0 \\ -2 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} -1 & 2 & 3 \\ 4 & 0 & 1 \end{pmatrix}$.

(a) Calculate $(AB)C$.

(b) Calculate $A(BC)$.

(6) Calculate

$$\begin{pmatrix} 2+i & 1+2i \\ i & 3+i \end{pmatrix} \begin{pmatrix} 2i & 2+i \\ 1+i & 1+2i \end{pmatrix}$$

where i is the complex number i .

(7) Calculate

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}.$$

(8) Calculate

(a)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

(b)

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

(c)

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

(9) Find the transposes of each of the following matrices.

$$(a) \ A = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ -1 & 1 \end{pmatrix}.$$

$$(b) \ B = \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -4 \\ 3 & 2 & 0 \end{pmatrix}.$$

$$(c) \ C = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}.$$

(10) Calculate

$$\begin{pmatrix} 2 & 0 \\ 7 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 3 & 3 \end{pmatrix}.$$

(11) Calculate

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ -4 \end{pmatrix} \begin{pmatrix} 3 & 1 & 5 \end{pmatrix}.$$

(12) Calculate A^2 , A^3 and A^4 where

$$A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}.$$

(13) Calculate $A\mathbf{x}$, $A^2\mathbf{x}$, $A^3\mathbf{x}$, $A^4\mathbf{x}$ and $A^5\mathbf{x}$ where

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \mathbf{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

What do you notice?

(14) Calculate

$$\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}$$

and simplify.

(15) Calculate $A^3 - 5A^2 + 8A - 4I$ where

$$A = \begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$$

and I is the 3×3 identity matrix.