Exercises 1

(1) Here are two puzzles by Raymond Smullyan, mathematician and magician. On an island there are two kinds of people: *knights* who always tell the truth and *knaves* who always lie. They are indistinguishable.

  (a) You meet three such inhabitants A, B and C. You ask A whether he is a knight or knave. He replies so softly that you cannot make out what he said. You ask B what A said and they say 'he said he is a knave'. At which point C interjects and says 'that's a lie!'. Was C a knight or a knave?

  (b) You encounter three inhabitants: A, B and C.
  A says 'exactly one of us is a knave'.
  B says 'exactly two of us are knaves'.
  C says: 'all of us are knaves'.
  What type is each?

(2) This question is a variation of one that has appeared in the puzzle sections of many magazines. There are five houses, from left to right, each of which is painted a different colour, their inhabitants are called Sarah, Charles, Tina, Sam and Mary, but not necessarily in that order, who own different pets, drink different drinks and drive different cars.

  (a) Sarah lives in the red house.
  (b) Charles owns the dog.
  (c) Coffee is drunk in the green house.
  (d) Tina drinks tea.
  (e) The green house is immediately to the right (that is: your right) of the white house.
  (f) The Oldsmobile driver owns snails.
  (g) The Bentley owner lives in the yellow house.
  (h) Milk is drunk in the middle house.
  (i) Sam lives in the first house.
  (j) The person who drives the Chevy lives in the house next to the person with the fox.
  (k) The Bentley owner lives in a house next to the house where the horse is kept.
  (l) The Lotus owner drinks orange juice.
  (m) Mary drives the Porsche.
  (n) Sam lives next to the blue house.

There are two questions: who drinks water and who owns the aardvark?

(3) *Bulls and Cows* is a code-breaking game for two players: the code-setter and the code-breaker. The code-setter writes down a 4-digit secret number all of whose digits must be different. The code-breaker tries to guess this number. For each guess they make, the code-setter scores their answer: for each digit in the right position score 1 bull (1B), for each correct digit in the wrong position score 1 cow (1C); no digit is scored twice. The goal is to guess the secret number in the smallest number of guesses. For example, if the secret number is 4271 and I guess 1234 then my score will be 1B,2C. Here's an easy problem. The following is a table of guesses and scores. What are the possibilities for the secret number?

1389	$0B, 0C$
1234	$0B, 2C$
1759	$1B, 1C$
1785	$2B, 0C$

(4) Consider the following algorithm. The input is a positive whole number n; so $n = 1, 2, 3, \ldots$ If n is even, divide it by 2 to get $\frac{n}{2}$; if n is odd, multiply it by 3 and add 1 to get $3n+1$. Now repeat this process and only stop if you reach 1. For example, if $n = 6$ we get successively $6, 3, 10, 5, 16, 8, 4, 2, 1$ and the algorithm stops at 1. What happens if $n = 11$? What about $n = 27$? Is it true that whatever whole number you input this procedure always yields 1?

(5) *Hofstadter's MU-puzzle.* A *string* is just an ordered sequence of symbols. In this puzzle, you will construct strings using the letters M, I, U where each letter can be used any number of times, or not at all. You are given the string MI which is your only input. You can make new strings only by using the following rules any number of times in succession in any order:

(I): If you have a string that ends in I then you can add a U on at the end.

(II): If you have a string Mx where x is a string then you may form Mxx.

(III): If III occurs in a string then you may make a new string with III replaced by U.

(IV): If UU occurs in a string then you may erase it.

I shall write $x \to y$ to mean that y is the string obtained from the string x by applying one of the above four rules. Here are some examples:

- By rule (I), $MI \to MIU$.
- By rule (II), $MIU \to MIUIU$.
- By rule (III), $UMIIIMU \to UMUMU$.
- By rule (IV), $MUUUII \to MUII$.

The question is: can you make MU?

(6) *Sudoku puzzles* have become very popular in recent years. The newspaper that first launched them in the UK went to great pains to explain that they had nothing to do with maths despite involving numbers. Instead, they said, they were logic problems. This of course is nonsense: logic is part of mathematics. What they should have said is that they had nothing to do with *arithmetic*. The goal is to insert digits in the boxes to satisfy two conditions: first, each row and each column must contain all the digits from 1 to 9 exactly once, and second, each 3×3 box must contain the digits 1 to 9 exactly once.

	1		4	2				5
		2		7	1		3	9
							4	
2		7	1					6
				4				
6					7	4		3
	7							
1	2		7	3		5		
3				8	2		7	

(7) In this question, you may assume that the sum of the angles in a triangle is $180°$. Prove that the sum of the interior angles in any quadrilateral is equal to $360°$.

(8) In this question, you may assume Pythagoras' theorem.

(a) A rectangular box has sides of length 2, 3 and 7 units. What is the length of the longest diagonal?

(b) Draw a square. Without measuring any lengths, construct a square that has exactly twice the area.

(c) A right-angled triangle has sides with lengths x, y and hypotenuse z. Prove that if the area of the triangle is $\frac{z^2}{4}$ then the triangle is isosceles.

(9) You have an unlimited supply of 3 cent stamps and an unlimited supply of 5 cent stamps. By combining stamps of different values you can make up other values: for example, three 3 cent

stamps and two 5 cent stamps make the value 19 cents. What is the largest value you *cannot* make?