Exercises 2

(1) Let $A = \{\clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$, $B = \{\spadesuit, \diamondsuit, \clubsuit, \heartsuit\}$ and $C = \{\spadesuit, \diamondsuit, \clubsuit, \heartsuit, \clubsuit, \diamondsuit, \heartsuit, \spadesuit\}$. Is it true or false that $A = B$ and $B = C$? Explain.

(2) Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Write down the following subsets of X.
 (a) The subset A of even elements of X.
 (b) The subset B of odd elements of X.
 (c) $C = \{x\colon\ x \in X \text{ and } x \geq 6\}$.
 (d) $D = \{x\colon\ x \in X \text{ and } x > 10\}$.
 (e) $E = \{x\colon\ x \in X \text{ and } x \text{ is prime}\}$.
 (f) $F = \{x\colon\ x \in X \text{ and } (x \leq 4 \text{ or } x \geq 7)\}$.

(3) (a) Find all subsets of $\{a, b\}$. How many are there? Write down also the number of subsets with, respectively, 0, 1 and 2 elements.
 (b) Find all subsets of $\{a, b, c\}$. How many are there? Write down also the number of subsets with, respectively, 0, 1, 2 and 3 elements.
 (c) Find all subsets of the set $\{a, b, c, d\}$. How many are there? Write down also the number of subsets with, respectively, 0, 1, 2, 3 and 4 elements.
 (d) What patterns do you notice arising from these calculations?

(4) Write down the elements of the set $\{A, B, C\} \times \{a, b\}$.

(5) If the set A has m elements and the set B has n elements how many elements does the set $A \times B$ have?

(6) If A has m elements, how many elements does the set A^n have?

(7) Let $S = \{4, 7, 8, 10, 23\}$, $T = \{5, 7, 10, 14, 20, 25\}$ and $V = \{2, 5, 10, 20, 30, 36\}$. Determine the following.
 (a) $S \cup (T \cap V)$.
 (b) $S \setminus (T \cap V)$.
 (c) $(S \cap T) \setminus V$.

(8) Let $A = \{a, b, c, d, e, f\}$ and $B = \{g, h, k, d, e, f\}$. What are the elements of the set $A \setminus ((A \cup B) \setminus (A \cap B))$?

(9) Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$. What is the set

$$(A \times B) \setminus ((\{1\} \times B) \cup (A \times \{c\}))?$$

(10) Prove that two sets A and B are equal if and only if $A \subseteq B$ and $B \subseteq A$.

(11) Given a set A define a new set $A^+ = A \cup \{A\}$. Calculate in succession the sets

$$\emptyset^+, \emptyset^{++}, \emptyset^{+++}$$

which are obtained by repeated application of the operation $+$. Write down the cardinalities of these sets.