Exercises 3

(1) None of these questions is intrinsically interesting. The point is to find the correct translation from the 'real-life' problem to the corresponding mathematical counting problem.
   (a) A menu consists of 2 starters, 3 main courses and 4 drinks. How many possible dinners are there consisting of one starter, one main course and one drink?
   (b) For the purposes of this question, a *date* consists of an ordered triple consisting of the following three components: first component a natural number d in the range $1 \leq d \leq 31$; second component a natural number m in the range $1 \leq m \leq 12$; third component a natural number y in the range $0 < y \leq 3000$. How many possible dates are there?
   (c) In how many ways can 10 books be arranged on a shelf?
   (d) There are 10 contestants in a race. Assuming no ties, how many possible outcomes of the race are there?
   (e) 8 cars are to be ranked first, second and third. In how many ways can this be done?
   (f) In how many ways can a hand of 13 cards be chosen from a pack of 52 cards?
   (g) In how many ways can a committee of 4 people be chosen from 10 candidates?
   (h) A committee of 9 people has to elect a chairman, secretary and treasurer (assumed all different). In how many ways can this be done?
   (i) In a lottery, 6 distinct numbers are chosen from the range 1 to 49 without regard for order. In how many ways can this be done?
   (j) Given the digits 1,2,3,4,5 how many 4-digit numbers can be formed if repetition is allowed?

(2) A novel has 250 pages, each page has 45 lines, and each line consists of 60 symbols. Assume that the number of possible symbols available is 100. How many possible novels are there? We allow avant garde novels that consist of nonsense words or are blank.[1] Express the answer as a power of ten.

(3) (a) Let A and B be finite sets. Prove that

$$|A \cup B| = |A| + |B| - |A \cap B| \,.$$

   (b) Generalize your result to obtain a similar formula for $|A \cup B \cup C|$.

[1] Or do not contain the letter 'e' such as the novel *Gadsby* by Ernest Vincent Wright.