Exercises 4

(1) Prove the following using the axioms for $\mathbb{R}$.
  (a) $(a + b)^2 = a^2 + 2ab + b^2$.
  (b) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
  (c) $a^2 - b^2 = (a + b)(a - b)$.
  (d) $(a^2 + b^2)(c^2 + d^2) = (ac - bd)^2 + (ad + bc)^2$.

(2) Let $a, b, c \in \mathbb{R}$. If $ab = ac$ is it true that $b = c$? Explain.

(3) Calculate the discriminants of the following quadratics and so determine whether they have distinct roots, repeated roots, or no real roots.
  (a) $x^2 + 6x + 5$.
  (b) $x^2 - 4x + 4$.
  (c) $x^2 - 2x + 5$.

(4) Solve the following quadratic equations by completing the square.
  (a) $x^2 + 10x + 16 = 0$.
  (b) $x^2 + 4x + 2 = 0$.
  (c) $2x^2 - x - 7 = 0$.

(5) I am thinking of two numbers x and y. I tell you their sum a and their product b. What are x and y in terms of a and b?

(6) Let $p(x) = x^2 + bx + c$ be a monic quadratic with roots x_1 and x_2. Express the discriminant of $p(x)$ in terms of x_1 and x_2.