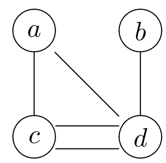
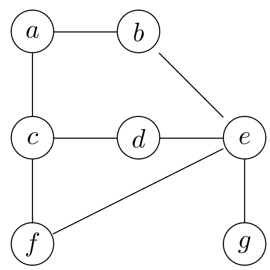
Exercises 1

1. Write down the number of vertices and the number of edges of each of the following graphs.

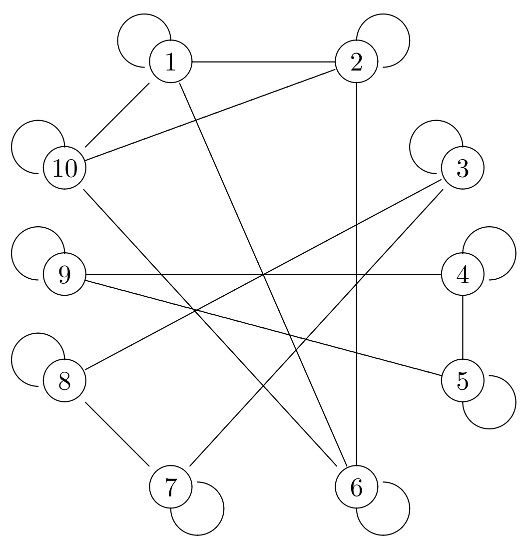
   (a)



   (b)



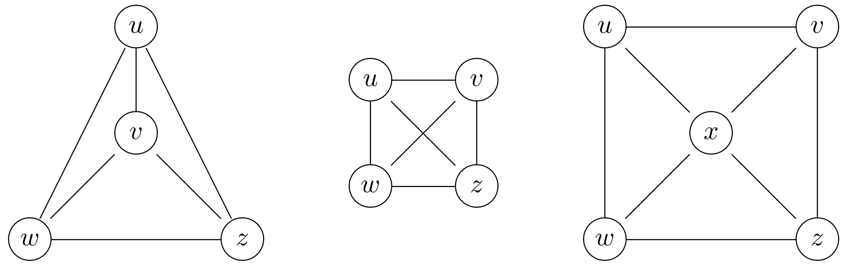
   (c)



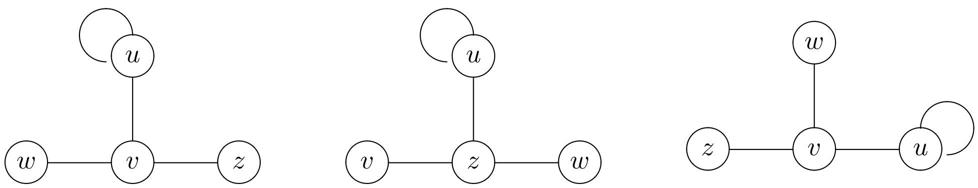
   For each of the graphs above, write down the degrees of each vertex. Compare the sum of all the degrees of each graph with the number of its edges. What do you notice?

2. In each of the following, two of the graphs are the same and the third is different. Find the odd man out in each case.

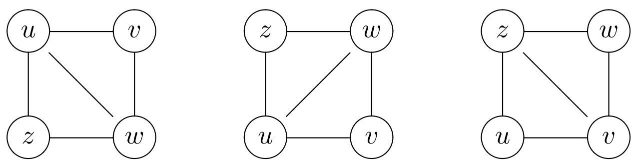
(a)



(b)

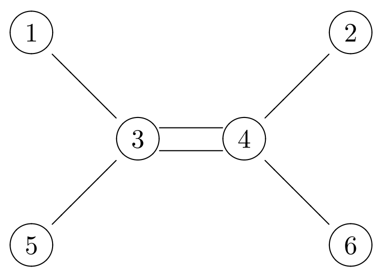


(c)

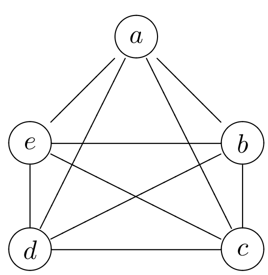


3. Which of the following graphs (a) contain multiple edges (b) contain a loop (c) are simple (d) are connected (e) contain an isolated vertex?

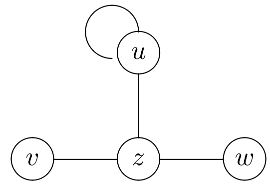
(1)



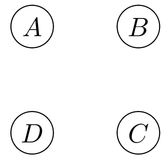
(2)



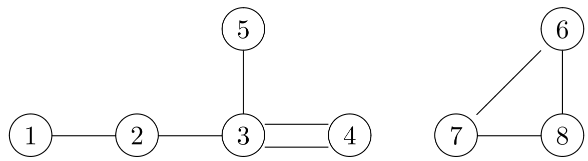
(3)



(4)



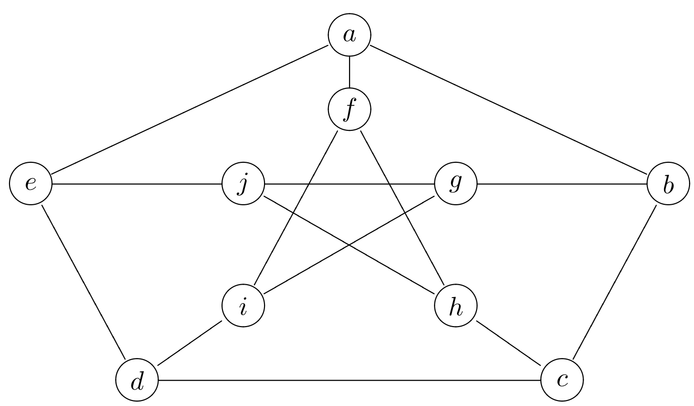
(5)



For each graph above, write down its degree sequence.

4. Find all simple unlabelled graphs with 4 vertices.

5. Find cycles in the Petersen graph (below) of lengths 5,6,8 and 9 respectively.



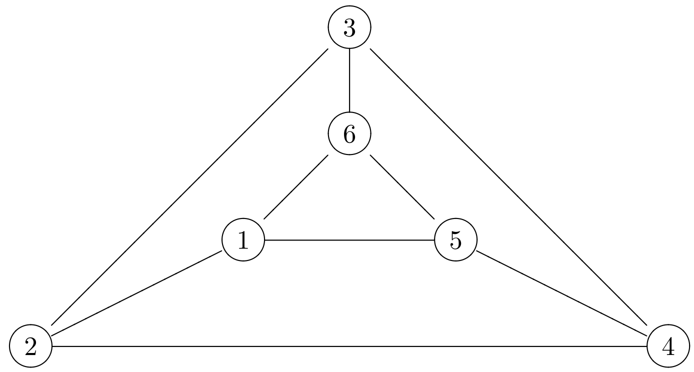
6. Find the number of vertices and edges in each of the following graphs:

   (i) The null graphs N_n.

   (ii) The cycle graphs C_n.

   (iii) The complete graphs K_n.

(iv) The complete bipartite graphs $K_{m,n}$.

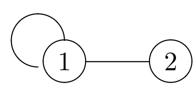


7. A *binary string* is a finite sequence of 0's and 1's. The *length* of a binary string is the total number of symbols occurring in it.

   (i) Draw the following graph: the vertices are labelled by binary strings of length 3 (i.e. all possible sequences of three 0's and 1's from 000 to 111); two vertices are joined by an edge when they differ in exactly once place. Thus 000 is joined to 100 but not to 110.

   (ii) Repeat the question for binary strings of length 4. (Remark: graphs such as these are very important in the design of 'error-correcting codes' such as those used in CD's).

8. In the graph below, find the total number of paths from vertex 1 to itself of lengths 1,2,3,4,5,and 6 respectively. What pattern do you notice in the numbers you obtain?



9. Draw the following graphs:

   (a) A simple graph with 5 vertices and 6 edges.

   (b) Two different regular graphs with 5 vertices.

   (c) A simple graph with degree-sequence $(2, 2, 2, 3, 3)$

   (d) A graph with degree-sequence $(2, 2, 2, 2, 3)$

   (e) Two different graphs with 6 vertices, 9 edges and degree sequence $(2, 2, 3, 3, 3, 5)$

10. Prove that the number of vertices of odd degree in a graph must be even.

11. Let G be a simple graph with at least two vertices. Prove that G must contain at least two vertices of the same degree.