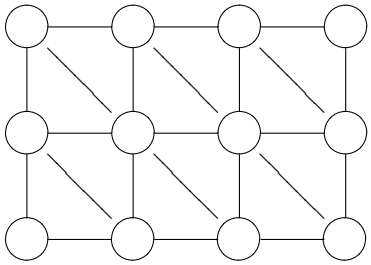
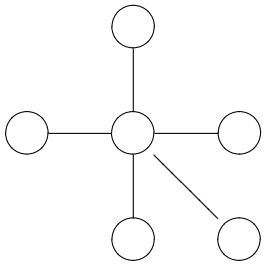
Exercises 4

1. For each of the planar graphs below find: the number of vertices (v), the number of edges (e), and the number of faces (f). In each case, calculate $f - e + v$.

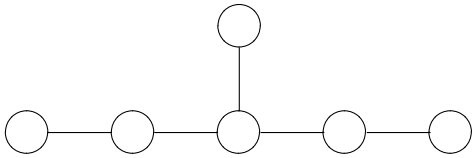
   (i)



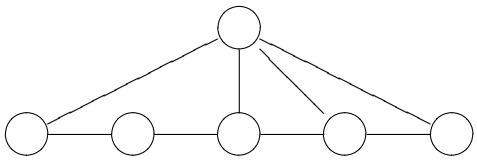
   (ii)



   (iii)

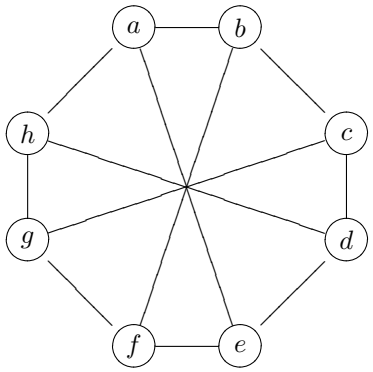


   (iv)



2. For each of the graphs in Question 1, calculate the face-degrees of each face, and the sum of all the face degrees — what is the sum of all the face degrees equal to, and why?

3. Use Kuratowski's Theorem to prove that the following graph is not planar.



4. Are all trees planar?

5. Which of the graphs K_n are planar and which non-planar. Explain.

6. Which of the graphs $K_{m,n}$ are planar and which non-planar. Explain.

7. A *fullerene* is a planar graph in which each vertex has degree 3 and where all faces are 5-cycles or 6-cycles. Prove that a fullerene has exactly twelve 5-cycles.[1]

8. The *girth* of a graph is the length of the smallest cycle in the graph. Let G be a simple connected planar graph with $v \geq 3$ vertices, e edges, and girth g. Prove that
$$e \leq \frac{g}{g-2}(v-2).$$

[1] This is my favourite question in the whole course. If you want to know what a 'fullerene' is I recommend Jim Baggott's book *Perfect symmetry: the accidental discovery of Buckminsterfullerene*, OUP, 1996.