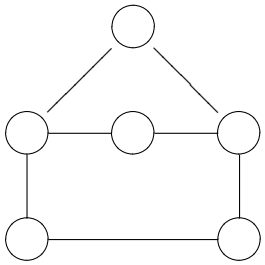
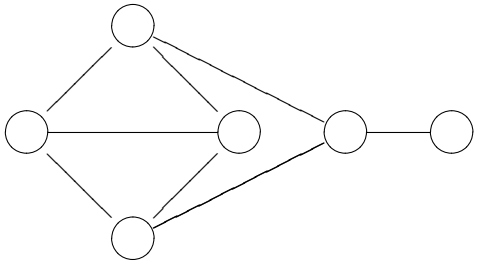
Exercises 5

1. Find the chromatic numbers of each of the following graphs and justify your answers.

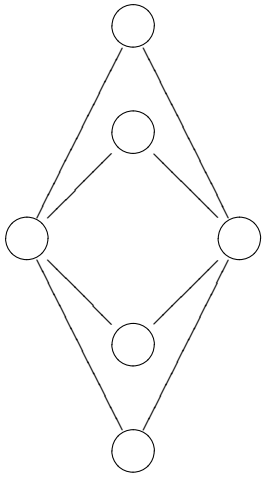
   (i)



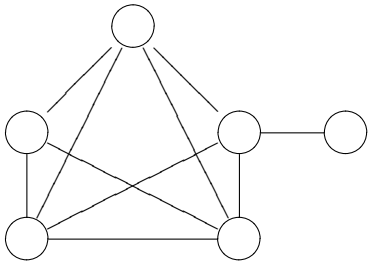
   (ii)



   (iii)



(iv)



2. A school plans to offer seven courses next year in: Maths (M); English (E); French (F); History (H); Science (S); Geography (G); RE (R). There are 12 pupils who make the following choices:

   Adam (M), (S), (F).

   Beth (M), (S), (G).

   Charles (E), (F).

   Delia (M), (E), (S).

   Ed (E), (R).

   Ffion (E), (H).

   George (M), (S).

   Hariot (E), (F).

   Ifor (E), (R).

   Jane (M), (S), (G).

   Kelvin (G), (R).

   Lois (E), (F).

   The problem is to find the minimum number of periods needed to timetable these courses without clashes.

   (i) Represent this information by means of a suitable graph, and so explain why this problem can be regarded as a vertex-colouring problem.

   (ii) Find the chromatic number of the graph and so find the minumum number of periods needed. Justify your answer.

3. Which graphs have chromatic number 1?

4. Which graphs have chromatic number 2?

5. What are the chromatic numbers of the graphs K_n?

6. What are the chromatic numbers of the graphs $K_{m,n}$?

7. What are the chromatic numbers of trees having at least two vertices?

8. Suppose that we have 6 chemicals and, for each i from 1 to 4, chemical i cannot be stored in the same room as chemicals $i+1$ or $i+2$. Turn this into a graph colouring problem and so find the smallest number of rooms needed for the safe storage of these chemicals.

9. If a graph contains a triangle (in other words a cycle of length 3) then its chromatic number is ≥ 3. Is it true that a graph whose chromatic number is ≥ 3 must contain a triangle somewhere as a subgraph?

10. Prove that a graph has chromatic number 2 if and only if it does not contain any cycles of odd length. (Hard!)