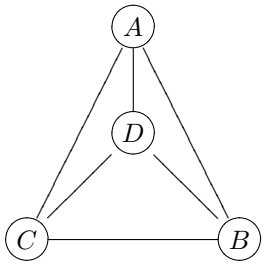
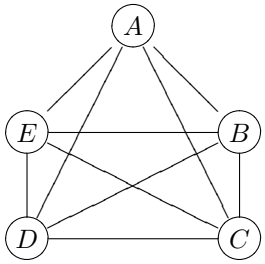
Exercises 6

1. For each of the graphs below determine whether it is Eulerian or not with reasons. For those that are Eulerian find an Euler tour by inspection.

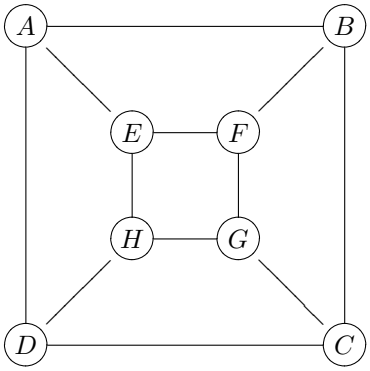
   (a)



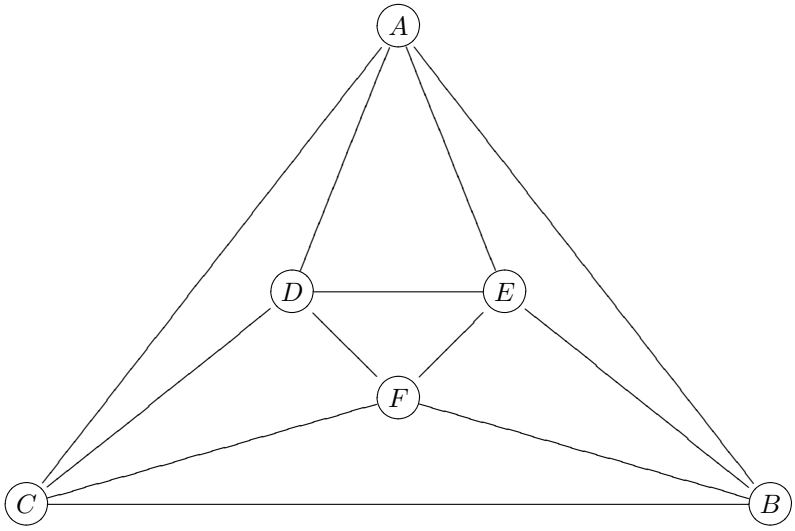
   (b)



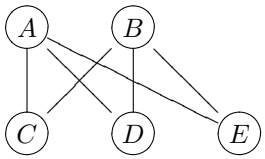
   (c)



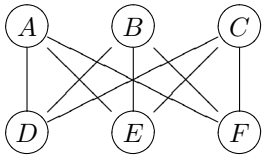
(d)



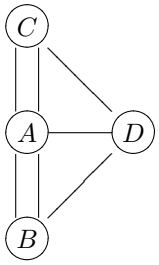
(e)



(f)

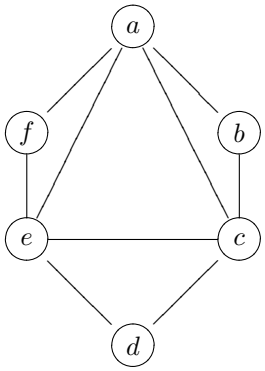


(g)



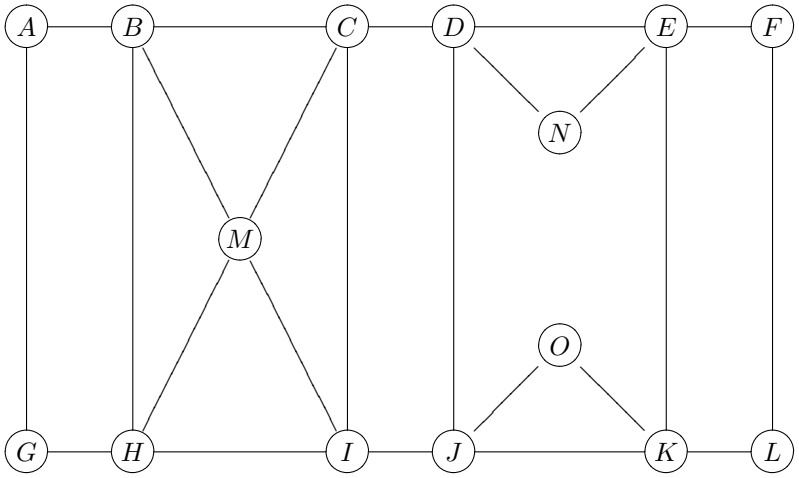
2. Show that the Eulerian graph below can be split up into cycles no two of which have any edges in common. There are 5 possible solutions; find

them all.

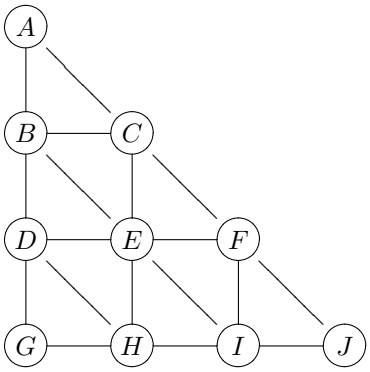


3. Use the algorithm described in the lectures to find an Euler tour of each of the following Eulerian graphs.

   (i)



   (ii)



4. For which values of n is the graph K_n Eulerian, and why.

5. For which values of m and n is the graph $K_{m,n}$ Eulerian, and why.

6. A graph is said to be *edge-traceable* if there is a path beginning and ending at different vertices which uses every edge exactly once. Give an example of an edge-traceable graph which is not Eulerian. Prove that a connected graph is edge-traceable if, and only if, it has exactly two vertices of odd degree. (Hint: what happens if two vertices of odd degree are joined by a new edge?).