Solutions: Exercises 6

1. (a) Not Eulerian — has vertex of odd degree.
   (b) Eulerian — every vertex has even degree and graph is connected. Euler tour: AEDCBDACEBA. (I would want you to draw the tour).
   (c) Not Eulerian — has vertex of odd degree.
   (d) Eulerian — every vertex has even degree and graph is connected. Euler tour: ADEABEFBCDFCA.
   (e) Not Eulerian — vertex of odd degree.
   (f) Not Eulerian — vertex of odd degree.
   (g) Not Eulerian — vertex of odd degree.

2. There are 5 sets of complete disjoint cycles: abca, cdec, aefa; abcdefa, acea; abcefa, acdea; abcdea, acef; acdefa, abcea.

3. (i) One set of cycles is: ABCDEFLKJIHGA, BMHB, CIMC, DNEKOJD. The resulting Euler tour is: ABMHBCIMCDNEKOJDEFLKJIHGA.
   (ii) One set of cycles is: ACFJIHGDBA, BCEB, EFIE, DEHD. The resulting Euler tour is: ACEBCFIEFJIHDEHGDBA.

4. For n odd, because when n is odd each vertex is joined to $n-1$ others which is therefore an even number. It is clear that K_n is always connected.

5. Both m and n need to be even, because when n is even each top vertex is joined to n bottom vertices, and when m is even each bottom vertex is joined to m top vertices.

6. Suppose graph is connected and has exactly two vertices of odd degree u and v. Make a new graph by joining u and v by an edge e. The resulting graph is connected and every vertex has even degree. Thus the graph is Eulerian and has an Euler tour. Erasing the edge e from this Euler tour gives a path that joins u and v and takes in every edge exactly once. Conversely, suppose a graph is edge traceable from u to v. Adding an edge e between u and v completes the path that witnesses the edge traceability of the graph into an Euler tour. It follows that each vertex has even degree. Thus u and v have even degree in the new graph and so, erasing e, odd degree in the new graph — all other vertices have even degree.