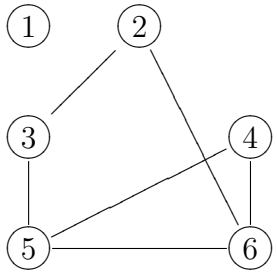
Solutions to Test 1/Algebra 3/2008

1. 28.
   Remember that where edges cross are not vertices.

2. 42.
   The graph has 28 vertices each of degree 3. Thus the sum of the vertex degrees is 84. By the Handshaking Lemma, the number of edges is half this number: namely, 42. You could, of course, just count them.

3. Many acceptable solutions here is one.

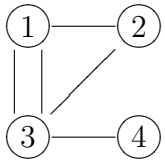


   This question is similar to Question 9 of Exercises 1.

4. Impossible.
   The sum of the numbers is odd, but by the Handshaking Lemma the sum of the vertex degrees is even. Thus the sequence of numbers cannot be a degree sequence. *This question is similar to Question 9 of Exercises 1.*

5.
$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

6.



7. 11,628.
   The graph has 153 vertices and each vertex has degree 152. By the Handshaking Lemma, if we multiply these two numbers together and divide by 2 we get the number of edges. Alternatively, it is just 153 choose 2. *This question is a special case of Question 6, Exercises 1.*

8. 819.
   The adjacency matrix A of K_5 is

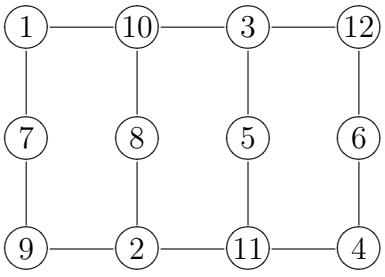
$$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix}$$

   We are interested in paths of length 6 and so we have to calculate A^6; this is just $A^2(A^2)^2$, for example. We therefore get

$$\begin{pmatrix} 820 & 819 & 819 & 819 & 819 \\ 819 & 820 & 819 & 819 & 819 \\ 819 & 819 & 820 & 819 & 819 \\ 819 & 819 & 819 & 820 & 819 \\ 819 & 819 & 819 & 819 & 820 \end{pmatrix}$$

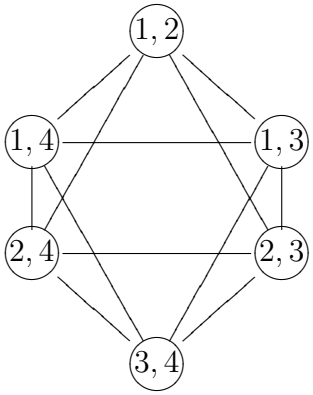
   The method needed to solve this question was one of the two topics discussed in Section 2; the other was induction, which I said you wouldn't be tested on ...

9. You have to draw the graph without edges crossing.



   You don't need to know chess to answer this question since the knight's move was described in the lectures in relation to Example 1.4.

10.



This is similar to Exercise 1.7 which was discussed in the lectures.