Properties of Boolean operations

This is a version of Section 3.2.

Key definitions

- Let X be a non-empty set. A *binary operation* $*$ on X associates with every ordered pair (x, y) of elements of X a single element $x * y$ of X.
- The binary operation $*$ is said to be *commutative* if $x * y = y * x$ for all $x, y \in X$.
- The binary operation $*$ is said to be *associative* if $(x * y) * z = x * (y * z)$ for all $x, y, z \in X$.
- An *identity* for $*$ is an element e such that $e * x = x = x * e$ for all $x \in X$.
- A *zero* for $*$ is an element z such that $z * x = z = x * z$ for all $x \in X$.
- The binary operation $*$ is said to be *idempotent* if $x * x = x$ for all $x \in X$.
- If $\bullet$ is another binary operation on the set X we say that $*$ *distributes over* $\bullet$ if $x * (y \bullet z) = (x * y) \bullet (x * z)$.

Properties

Let A, B and C be any sets.

(1) $A \cap (B \cap C) = (A \cap B) \cap C$. Intersection is associative.
(2) $A \cap B = B \cap A$. Intersection is commutative.
(3) $A \cap \emptyset = \emptyset = \emptyset \cap A$. The empty set is the zero for intersection.
(4) $A \cup (B \cup C) = (A \cup B) \cup C$. Union is associative.
(5) $A \cup B = B \cup A$. Union is commutative.
(6) $A \cup \emptyset = A = \emptyset \cup A$. The empty set is the identity for union.
(7) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Intersection distributes over union.
(8) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. Union distributes over intersection.
(9) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. De Morgan's law part one.
(10) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$. De Morgan's law part two.
(11) $A \cap A = A$. Intersection is idempotent.
(12) $A \cup A = A$. Union is idempotent.