Lecture 3

Set constructions

round brackets

Definition An ordered pair (a, b) has a as first component and b as second component. The key feature of ordered pairs is that $(a, b) = (c, d)$ iff $a = c$ & $b = d$.

In general, $(a, b) \neq (b, a)$ so order matters.

Definition Let A and B be sets. Then

$$A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$$

Called the product of A and B

<u>Example</u> Let $A = \{1, 2\}$ and ~~B~~

$B = \{a, b, c\}$. Then

$$A \times B = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$$

$$B \times A = \{ (a,1), (b,1), (c,1), (a,2), (b,2), (c,2) \}$$

Observe that $A \times B \neq B \times A$

In general,

If $A = B$, we write $A^2 = A \times A$

We can generalize all of the above.

<u>Definition</u> <u>ordered triples</u> (x, y, z)

. . . . <u>ordered n-tuples</u> $(x_1, \ldots, x_n)$.

Let $A_1, \ldots, A_n$ be ~~a~~ sets.

Then

$$A_1 \times \ldots \times A_n = \{ (a_1, \ldots, a_n) : a_1 \in A_1, \ldots, a_n \in A_n \}$$

If $A_1 = A_2 = \ldots = A_n$, we define

$$A^n = \underbrace{A \times \ldots \times A}_{n \text{ times}}.$$

Example

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, the real plane

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, real space.

Example Dates are really ordered triples.

(Bloomsday)

$$\begin{array}{ccc} (16, & \text{June}, & 1904) \\ \uparrow & \uparrow & \uparrow \\ \text{day} & \text{month} & \text{year} \end{array}$$

[Brit. Convention. US Convention reverses the first two components]

Example Probability theory uses the language of sets. For example, suppose two dice are thrown. The set of possible outcomes is

$$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

Ω is called the sample space. In probability theory, subsets of sample spaces are called events. If $X \subseteq \Omega$ then we define the probability of the event X to be

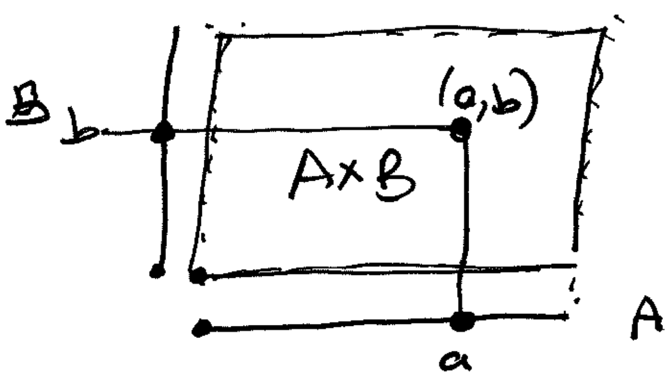
$$\frac{|X|}{|\Omega|}$$

So, to study probability, we need to be able to determine cardinalities of sets.

The event of throwing a double 6 is $\{(6,6)\}$. The probability of throwing a double 6 is therefore

$$\frac{|\{(6,6)\}|}{|\{1,2,3,4,5,6\}^2|} = \underline{\underline{\frac{1}{36}}}$$

<u>Example</u> What is $|A \times B|$?



$$|A \times B| = |A||B|$$

Boolean operations on sets

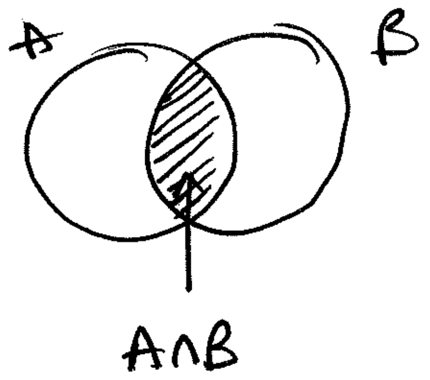
[From George <u>Boole</u>]

<u>Venn diagram</u> Represent the set A by a region in the plane.

<u>Definition</u> Let A and B be sets.

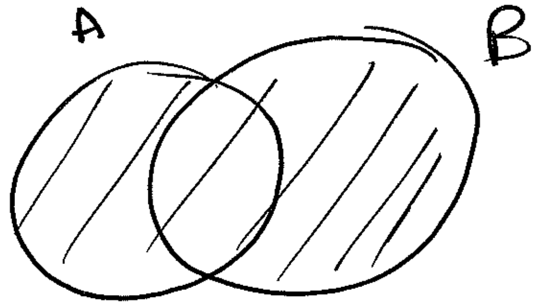
$$A \cap B = \{ a : a \in A \text{ and } b \in B \}$$

Called the <u>intersection</u> of <u>A and B</u>.

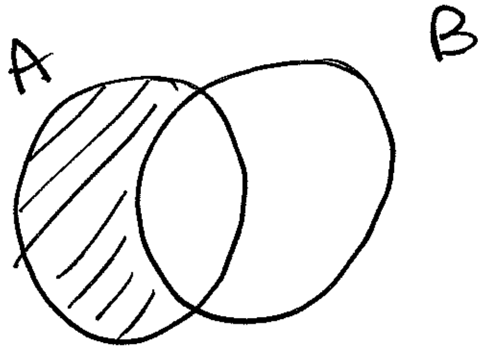


$A \cup B = \{a : a \in A \text{ or } a \in B \text{ or both}\}$

called the <u>union of A and B</u>.



$A \setminus B = \{a : a \in A \text{ and } a \notin B\}$



called the <u>relative complement of A and B.</u>

Examples

Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \setminus B = \{1, 2\}$

$B \setminus A = \{5, 6\}$

Definition Let A and B be sets.
If $A \cap B = \emptyset$, we say A and B are disjoint.