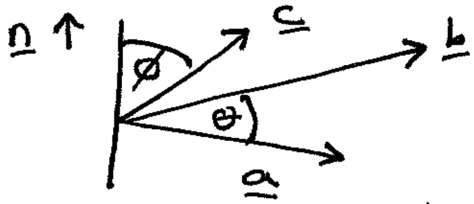
Lecture 25

Let $\underline{a}, \underline{b}, \underline{c}$ be vectors.



We calculate the volume of the parallelepiped det'd by $\underline{a}, \underline{b}, \underline{c}$.

- Area of base $= \|\underline{b}\| \sin\theta \, \|\underline{a}\|$
- Vertical height $= \|\underline{c}\| \cos\phi$

← absolute value.

$$\therefore \text{ volume } = \Big| \, \|\underline{a}\| \, \|\underline{b}\| \, \|\underline{c}\| \sin\theta \cos\phi \, \Big|$$

Observe that

$$\underline{a} \times \underline{b} = \left(\|\underline{a}\| \, \|\underline{b}\| \sin\theta \right) \underline{n}$$

$$\underline{c} \cdot (\underline{a} \times \underline{b}) = \|\underline{c}\| \, \|\underline{a}\| \, \|\underline{b}\| \sin\theta \cos\phi$$

$$\therefore \text{ volume of parallelepiped } = \left| \, \underline{c} \cdot (\underline{a} \times \underline{b}) \right|$$

$$\underline{c} \cdot (\underline{a} \times \underline{b}) = [\underline{c}, \underline{a}, \underline{b}]$$

$$= -[\underline{a}, \underline{c}, \underline{b}]$$

$$= [\underline{a}, \underline{b}, \underline{c}]$$

Interchanging two columns of a det changes its sign

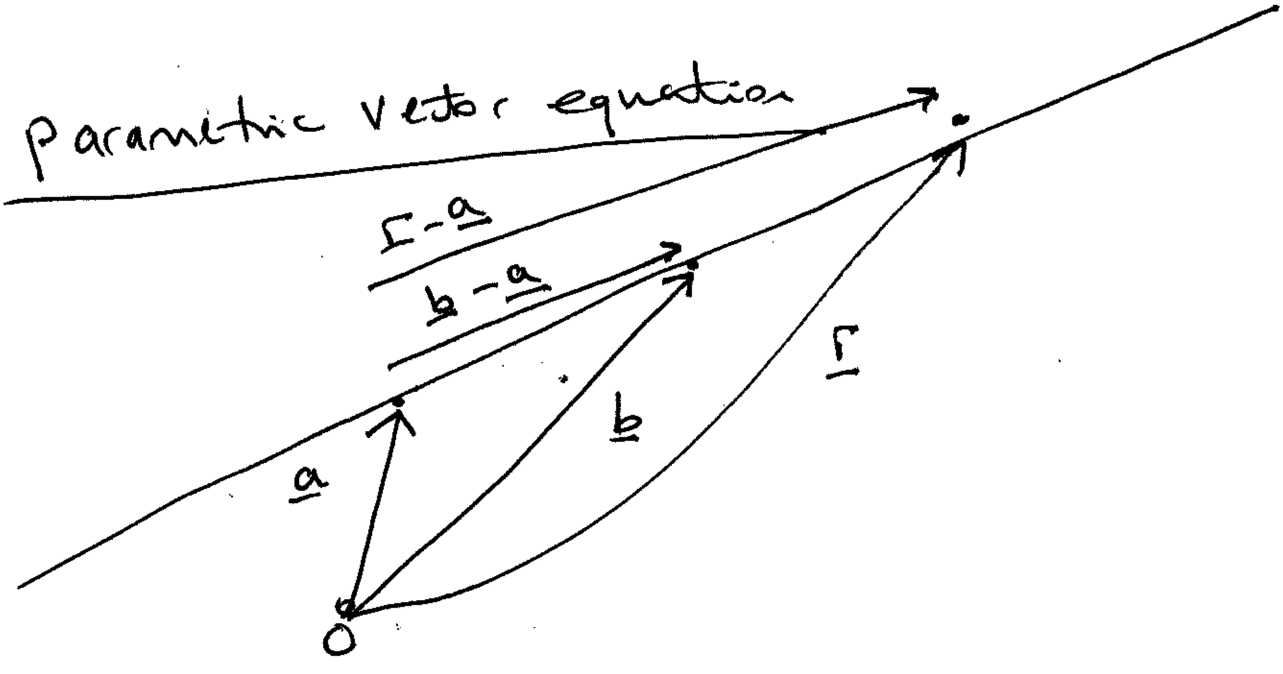
$\therefore$ det is signed volume. ∎

Vectors so far are <u>free vectors</u>.

A <u>position vector</u> is a vector rooted at a fixed point O (an <u>origin</u>). Position vectors enable us to describe specific points. The general position vector is denoted by $\underline{r}$ $(= x\underline{i} + y\underline{j} + z\underline{k})$.

Equation of the line

<u>parametric vector equation</u>



~~Ex~~ Two distinct points are described by means of two position vectors $\underline{a}$ and $\underline{b}$.

The vectors $\underline{r}-\underline{a}$ and $\underline{b}-\underline{a}$ $(\neq \underline{0})$ lie along parallel lines. Thus

$$\underline{r}-\underline{a} = \lambda(\underline{b}-\underline{a}) \quad \text{for some } \lambda \in \mathbb{R}.$$

It follows that

$$\boxed{\underline{r} = \underline{a} + \lambda(\underline{b}-\underline{a})}$$

<u>Remark</u> Observe that $\underline{c} = \underline{b}-\underline{a}$ is a vector parallel to the line. Thus the line is also defined by a point $\underline{a}$ and a vector $\underline{c}$ parallel to the line.

$$\underline{r} = \underline{a} + \lambda \underline{c}.$$

$$\therefore \quad x\underline{i} + y\underline{j} + z\underline{k} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k} + \lambda c_1\underline{i} + \lambda c_2\underline{j} + \lambda c_3\underline{k}$$

<u>Equate Components.</u>

$$\left.\begin{aligned} x &= a_1 + \lambda c_1 \\ y &= a_2 + \lambda c_2 \\ z &= a_3 + \lambda c_3 \end{aligned}\right\} \text{Cartesian parametric equations of the line.}$$

Now ~~to~~ eliminate λ.

$$\frac{x - a_1}{c_1} = \lambda \qquad (\text{if } c_1 \neq 0)$$

$$\frac{y - a_2}{c_2} = \lambda \qquad (\text{if } c_2 \neq 0)$$

$$\frac{z - a_3}{c_3} = \lambda \qquad (\text{if } c_3 \neq 0$$

$$\therefore \left\{ \begin{aligned} \frac{x-a_1}{c_1} &= \frac{y-a_2}{c_2} \\ \frac{y-a_2}{c_2} &= \frac{z-a_3}{c_3} \end{aligned} \right\}$$ non-parametric ~~eqn~~ Cartesian equations.

These are two linear equations (in x, y, z).

$$\left. \begin{aligned} c_2(x-a_1) &= c_1(y-a_2) \\ c_3(y-a_2) &= c_2(z-a_3) \end{aligned} \right\} \qquad \begin{aligned} c_2 x - c_1 y &= c_2 a_1 - c_1 a_2 \\ c_3 y - c_2 z &= c_3 a_2 - c_2 a_3 \end{aligned}$$