Lecture 26

Last time

Equation of the line

- Given two distinct position vectors $\underline{a}$ and $\underline{b}$ ~~eqn~~ parametric vector equation of the line they determine is

$$\boxed{\underline{r} = \underline{a} + \lambda(\underline{b} - \underline{a})}$$

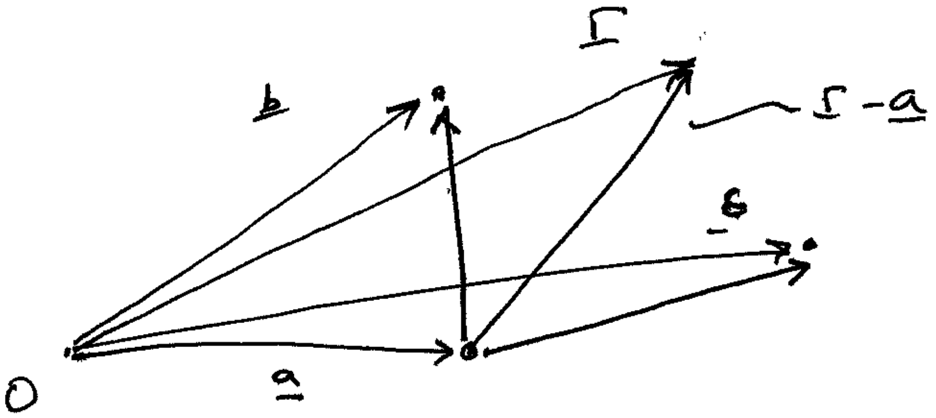
- Given one position vector $\underline{a}$ and a free vector $\underline{c}$ then the parametric vector equation of the line through $\underline{a}$ and parallel to $\underline{c}$ is

$$\boxed{\underline{r} = \underline{a} + \lambda \underline{c}}$$

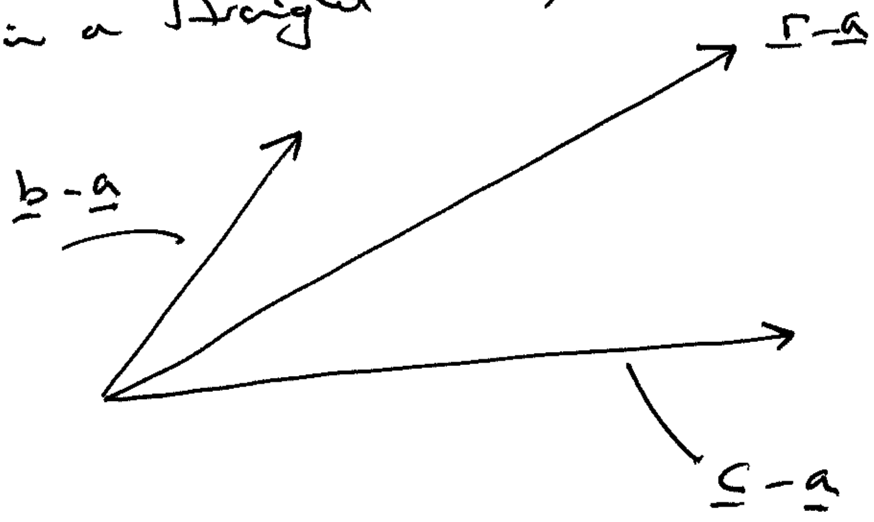
- Cartesian non-parametric equations of the line

$$\boxed{\begin{aligned} c_2 x - c_1 y &= c_2 a_1 - c_1 a_2 \\ c_3 y - c_2 z &= c_3 a_2 - c_2 a_3 \end{aligned}}$$

Equation of the plane



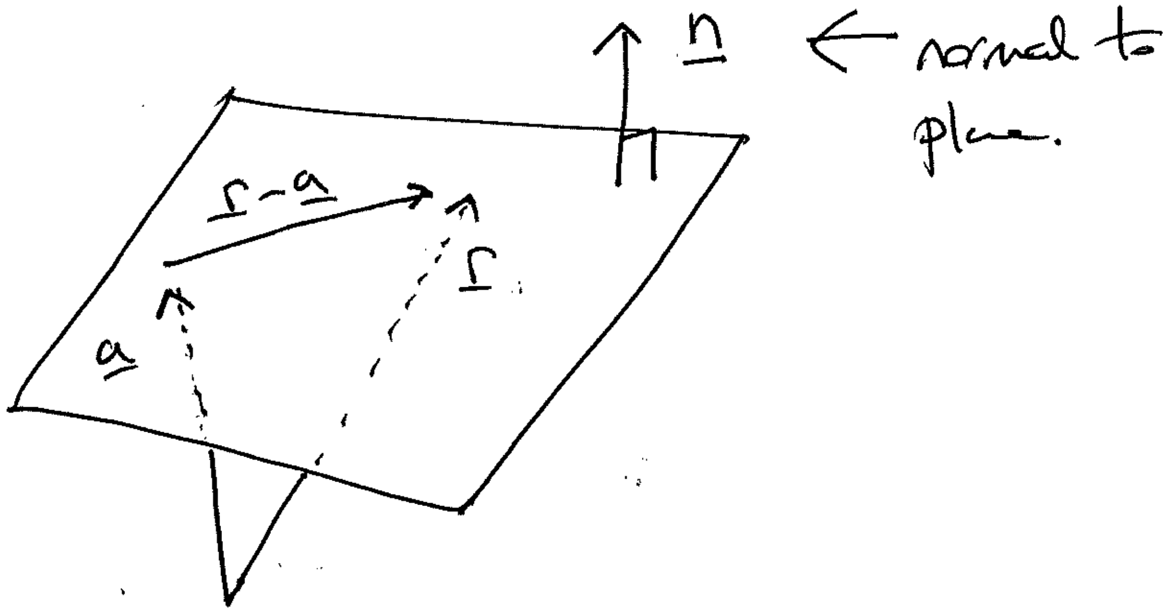
Choose three points with position vectors $\underline{a}, \underline{b}, \underline{c}$ (not all in a straight line)



$$\underline{r} - \underline{a} = \lambda(\underline{b} - \underline{a}) + \mu(\underline{c} - \underline{a}) \quad \text{where } \lambda, \mu \in \mathbb{R}$$

$$\therefore \quad \boxed{\underline{r} = \underline{a} + \lambda(\underline{b} - \underline{a}) + \mu(\underline{c} - \underline{a})}$$

parametric vector equation of the plane.



$$(\underline{r}-\underline{a})\cdot\underline{n}=0$$

$$\boxed{\underline{r}\cdot\underline{n}=\underline{a}\cdot\underline{n}}$$

→ Now we get the <u>non-parametric Cartesian equation</u>

$$\boxed{x n_1 + y n_2 + z n_3 = a_1 n_1 + a_2 n_2 + a_3 n_3}$$

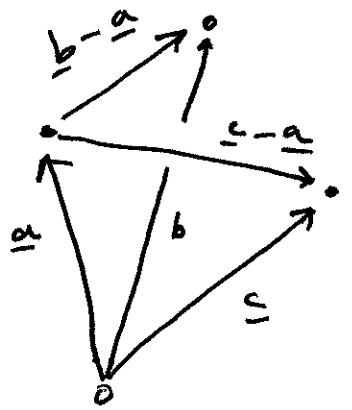
~~[illegible]~~

~~[illegible]~~ $\mathbb{R}^3$

~~Ex. It follows that the equation of the line is given by the intersection of two planes.~~

How to find a normal to a plane

Suppose the plane is determined by three points



The two vectors $\underline{b}-\underline{a}$ and $\underline{c}-\underline{a}$ are parallel to the plane. Then $(\underline{b}-\underline{a})\times(\underline{c}-\underline{a})$ is normal to the plane.

~~END OF SECTION~~

Systems of linear equations in 3 unknowns revisited

The equation

$$ax + by + cz = d$$

represents, in general, a plane in space.

Except when

- $a = b = c = d = 0$. When it is all of $\mathbb{R}^3$
- $a = b = c = 0$, $d \neq 0$ when it is $\emptyset$

Two such planes equations, in general, intersect in a line (~~this is the norm~~ thus giving the non-parametric Cartesian equations of the line)

However, they might be parallel and distinct and so ~~they~~ their intersection is $\emptyset$

Three such planes equations, in general, intersect in a point – but there are various other extreme possibilities.

What we are doing, when we use Erlos to solve systems of linear equations, is trying to determine whether each equation is bringing new information or old information (in disguised form).

End of section 4