Lecture 27

Proofs

Refer to Chapter 2 of my book

(1) The square of an even number is even.

Here number means a natural number. So, an element of $\mathbb{N}$. If $m \in \mathbb{N}$ then it is **even** if $2|m$ ('2 divides m' means $m = 2a$ for some $a \in \mathbb{N}$).

Let m be even.
Then $m = 2a$ for some $a \in \mathbb{N}$. Definition
Square both sides to get
$$m^2 = 4a^2 = 2(2a^2).$$
It follows that $2|m^2$ and so m^2 is even.

The square of an odd number is odd

$m \in \mathbb{N}$ is odd if $m = 2a+1$ for some $a \in \mathbb{N}$

Let m be an odd number.
Then $m = 2a+1$.
Square both sides to get

$$m^2 = 4a^2 + 4a + 1$$
$$= 2(2a^2 + 2a) + 1.$$

It follows that m^2 is odd.

m even $\Rightarrow$ m^2 even
m odd $\Rightarrow$ m^2 odd.

(2) We now prove the converses to the above two results.

m^2 even $\Rightarrow$ m even
m^2 odd $\Rightarrow$ m odd.