Lecture 28

Last time we proved the following two results:

(1) m even $\Rightarrow m^2$ even.
(2) m odd $\Rightarrow m^2$ odd.

These both have the form $A \Rightarrow B$.
The converse of $A \Rightarrow B$ is $B \Rightarrow A$
which may or may not be true.
We do prove the converse of (1) and (2)

(1)' m^2 even $\Rightarrow m$ even.
(2)' m^2 odd $\Rightarrow m$ odd.

Our proof of (1) and (2) were examples of direct proofs. To prove (1)' and (2)', we use indirect proofs.

m^2 even $\Rightarrow$ m even

Let m^2 be even.

Either m is odd or m is even. (Exhaustive mutually exclusive possibilities)

If m is odd then m^2 is odd by (1).

It follows that m is even.

[To prove statement S is true prove that the negation of S is false!]

Proof that m^2 odd $\Rightarrow$ m is odd is similar.

If $A \Rightarrow B$ and $B \Rightarrow A$ we write $A \Leftrightarrow B$ and say 'A if and only if B'.

Thus

$$\boxed{\begin{array}{l} m \text{ even} \Leftrightarrow m^2 \text{ even} \\ m \text{ odd} \Leftrightarrow m^2 \text{ odd} \end{array}}$$

Example

z is real $\Leftrightarrow \bar{z} = z$

<u>Application</u> $\mathbb{R} \setminus \mathbb{Q} \neq \emptyset$.

We prove, concretely, that $\sqrt{2} \in \mathbb{R} \setminus \mathbb{Q}$.

We use a technique called <u>proof by contradiction</u>.

If $a \in \mathbb{Q}$ then a is called <u>rational</u>

If $a \in \mathbb{R} \setminus \mathbb{Q}$ then a is called irrational = in-rational

= not-rational.

Either a is rational or a is irrational.

We show that a is rational is impossible.

This demonstrates that a is irrational.

Suppose that $\sqrt{2}$ is rational.

Then $\sqrt{2} = \frac{a}{b}$ where $a, b \in \mathbb{N}$.

Can assume that a and b have no common divisor (why?) (*)

Squaring both sides of above equation and rearranging

we get $2b^2 = a^2$.

Thus a^2 is even.

So, a is even. We can therefore write $a = 2x$

for some x. Thus $2b^2 = (2x)^2$ so

$b^2 = 2x^2$. Thus b^2 is even and so b is even.

It follows that a and b are both even contradicting (*)