

## QUIZ 1

(For feedback only will not contribute to your final grade)

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1. There are about 160 students in this class. How many ways are there of forming a student committee consisting of 4 students? You should explain how to model this question using set theoretic ideas.
2. The rank ordering for this course (who comes first, who comes second etc) will be decided by having the entire class (of 160 students) racing up Arthurs' Seat. Assuming no ties or fatalities, how many different outcomes are there? You should explain how to model this question using set theoretic ideas.
3. The *I Ching* (see Wikipedia if you don't know what this is) is based on pictures called *hexagrams*. Each hexagram consists of 6 horizontal lines which may either be line segments or broken line segments. Set up a bijection between the set of binary 6-tuples and the set of hexagrams and so determine how many hexagrams there are. You should make direct reference to any counting principles used.
4. Prove the following equality by counting **not** by algebra.

$$\binom{n}{k} = \binom{n}{n-k}.$$

5. Prove the following equality by counting **not** by algebra.

$$2^n = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n-2} + \binom{n}{n-1} + \binom{n}{n}.$$

6. This question establishes the connection between the numbers  $\binom{n}{k}$  and the numbers appearing in Pascal's triangle. Prove that

$$\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$$

where  $1 \leq r \leq n$ . You can use either algebra or counting to prove this equality (counting is quicker if you can spot how to do it). Explain the connection with Pascal's triangle.