Lecture 25

Quantification (We shall say more about this topic later - it is of fundamental importance).

$(\forall x)$: read 'for all x' (like $\wedge$)

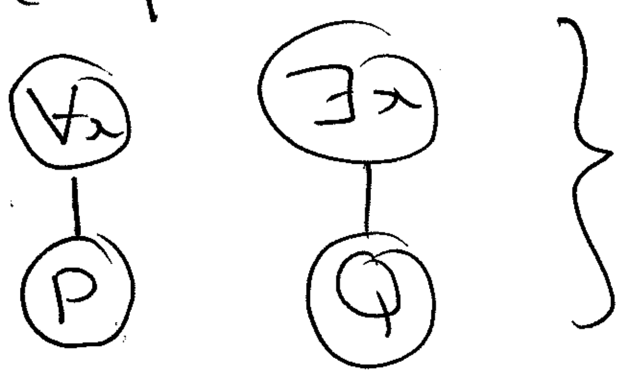
$(\exists x)$; read 'there exists an x' (like $\vee$)

= 'there exists at least one x'

Also, $(\forall x), (\forall y), (\forall z), \ldots$

$(\exists x), (\exists y), (\exists z), \ldots$

Regard quantifiers as unary connectives



parse trees.

$(\forall x)P$

The syntax of FOL

This consists of a choice of predicate symbols (with given arities) ~~(and constants)~~. In addition,

PL: $\neg, \vee, \wedge, \rightarrow, \leftrightarrow, \oplus$

Variables $x_1, x_2, x_3, \ldots$

Constants $a_1, a_2, a_3, \ldots$

Quantifiers $(\forall x_1), (\forall x_2), \ldots$
$(\exists x_1), (\exists x_2), \ldots$

Atomic formulae Predicate letters with variables and constants in all available slots.

Formula / wff

(F1) All atomic formulae are formulae.

(F2) If A and B are formulae so too are

$(\neg A)$, $(A \wedge B)$, $(A \vee B)$, $(A \to B)$,
$(A \leftrightarrow B)$, $(A \oplus B)$, $(\forall x) A$, $(\exists x) B$

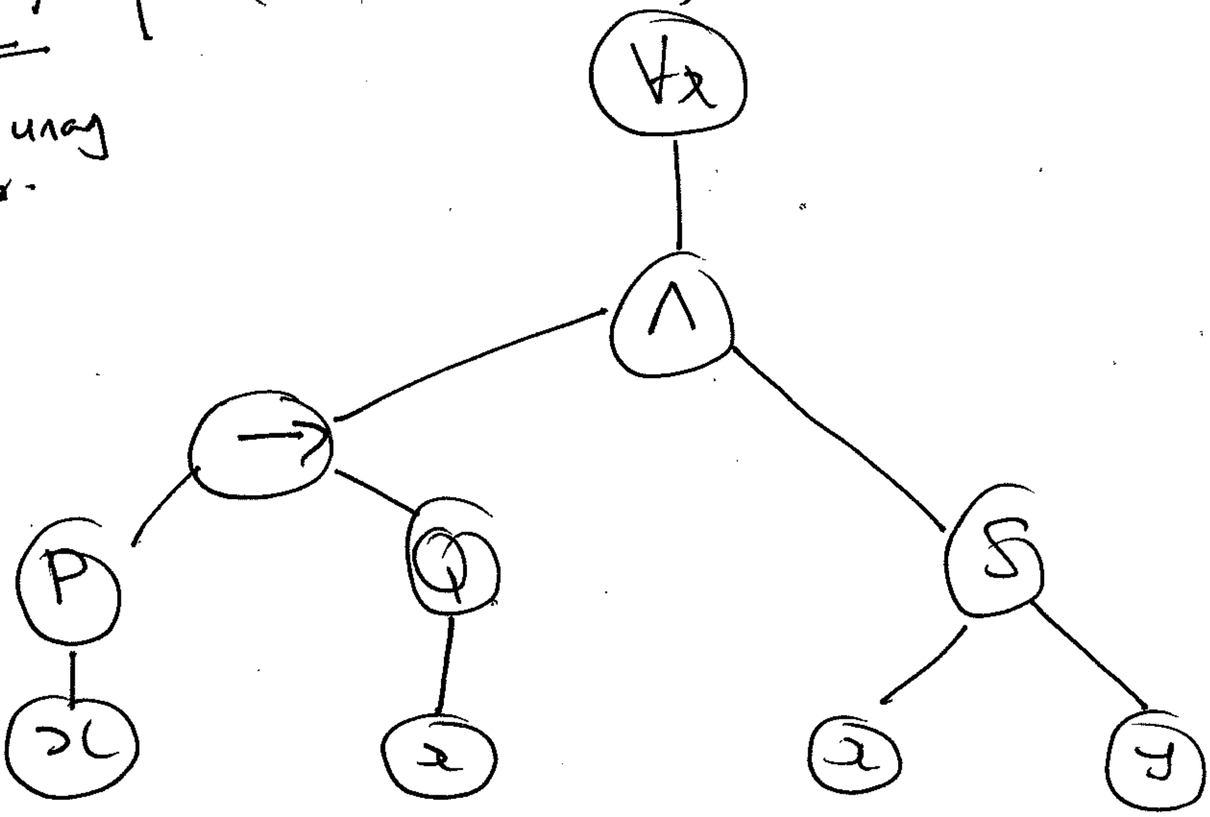
any variable   any variable.

F3) All formulae are obtained by a finite no. of applications of (F1) and (F2)

<u>Example</u> We draw the parse tree of the wff

$$\underline{(\forall x)} \left(\left(P(x) \to Q(x) \right) \land S(x, y) \right)$$

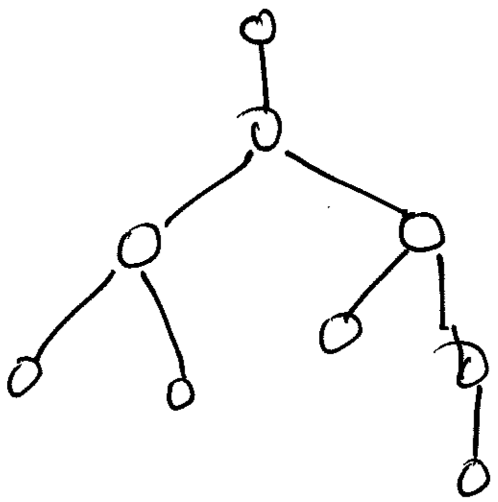
treat as unary operator.



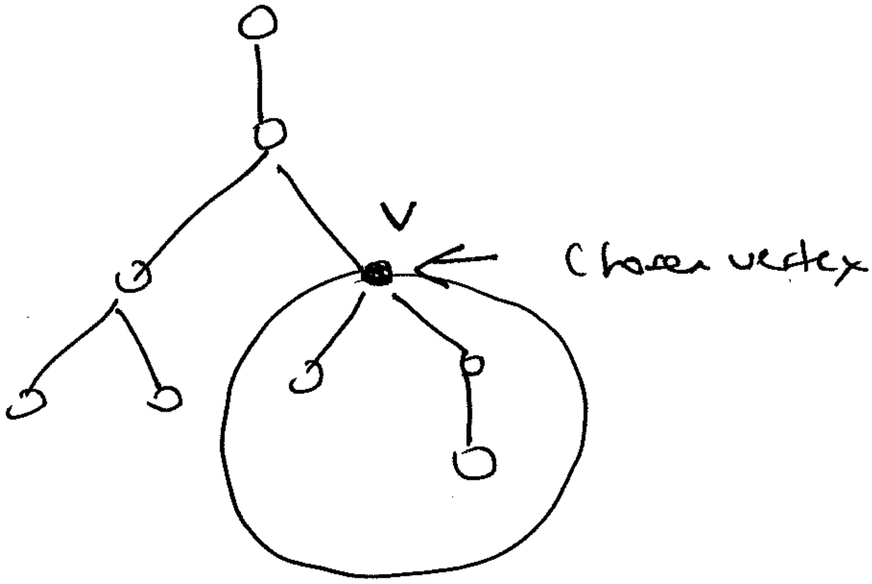
Unlike PL, we shall not study all wff.
We shall focus on those which we
Sentences. This is a technical term which
I shall now explain.

Subtrees

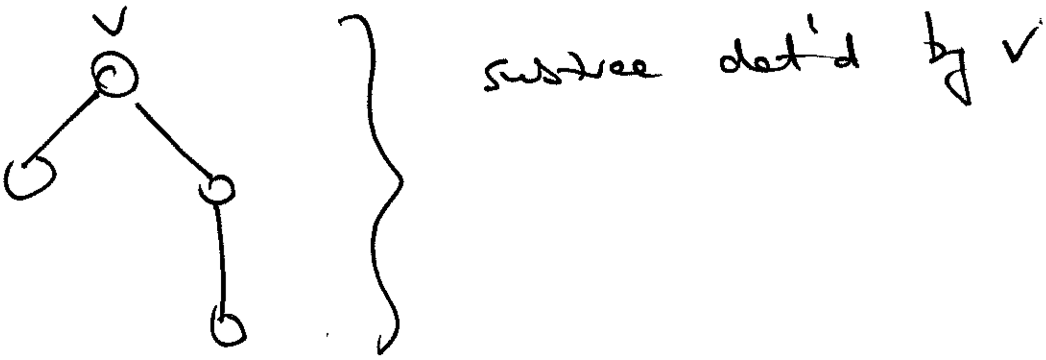
Consider the following tree



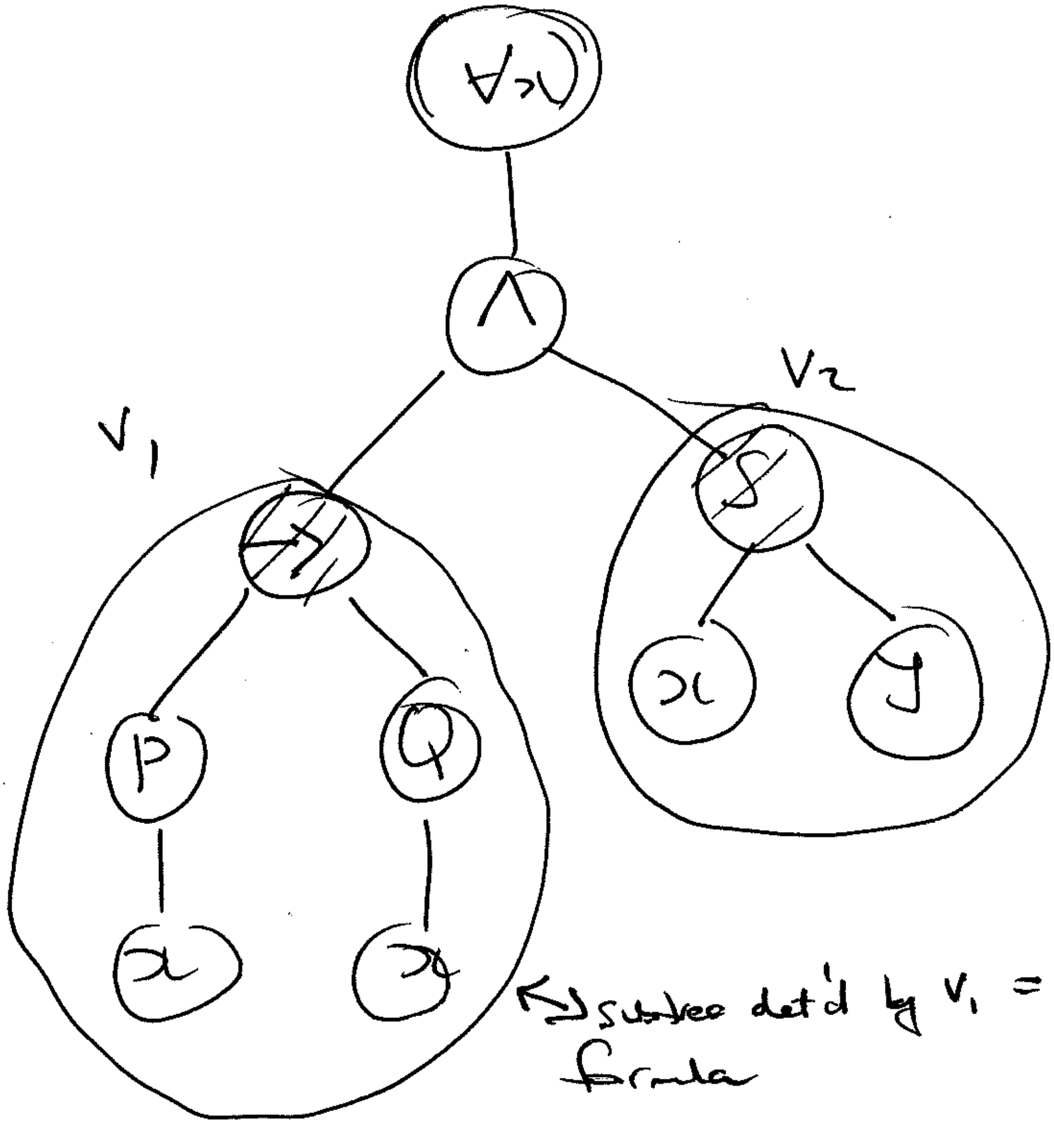
Choose any vertex which is not a leaf



Then this chosen vertex is itself the root of a tree, called the <u>subtree determined by v</u>



This idea of a subtree leads to the definition of a <u>subformula</u> of a wff.



subtree det'd by v_1 = parse tree of a formula

Subformula det'd by v_1 is

$$P(x) \to Q(x)$$

Subformula det'd by v_2 is

$$S(x, y)$$

Key definition Let A be a wff.

Let $(\forall x)$ [or $(\exists x)$] be an occurrence of $(\forall x)$ [or $(\exists x)$] in A.

This determines a subformula of A

$(\forall x)B$ [or $(\exists x)B$]. An occurrence of the variable x in $(\forall x)B$ [or $(\exists x)B$] is said to be **bound**.

> A sentence is a formula in which every variable is bound

Remark You should regard constants as being automatically bound.

Examples

(1) $(\forall x)\,[(P(x) \to Q(x)) \wedge S(x,y)]$

?

is not a sentence since y is not bound.

(= free).

(2) $(\forall x)\,(\forall y)\,[P(x) \to Q(y)]$

is a sentence because every ~~variable~~ occurrence of every variable is bound.

(3) $(\exists x)(F(x) \wedge G(x)) \rightarrow ((\exists x)F(x) \wedge (\exists x)G(x))$

This is a sentence because every occurrence of a variable is bound.

Why we focus on sentences will become clear when we discuss the semantics of FOL
