

Lecture 4

Set notation does not enable us to take account of order and repetition. To do this, we need to follow notation.

An ordered pair (a, b) has a first component and a second component.

An ordered triple and a third component.

(a_1, a_2, a_3) , an ordered quadruple (a_1, a_2, a_3, a_4) and a fourth component.

... more generally, an ordered n -tuple

(a_1, \dots, a_n) . The key feature of ordered pairs (or, more generally, ordered n -tuples) is that

$(a, b) = (c, d) \iff a = c \text{ and } b = d$

"if and only if"

$(a_1, \dots, a_n) = (b_1, \dots, b_n) \iff a_i = b_i \text{ for all } i$

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In general, $(a, b) \neq (b, a)$ unless $a = b$. Order matters.

(a, a) repetition allowed.

Let A and B be sets. Define the

Product of A and B , written $A \times B$, to be

the set

$$A \times B = \{ (a, b) : a \in A \text{ and } b \in B \}$$

Let $A = \{1, 2, 3\}$ and $B = \{a, b, c\}$.

Example

$$A \times B = \{ (1, a), (1, b), (1, c), (2, a), (2, b), (2, c), (3, a), (3, b), (3, c) \}$$

$$B \times A = \{ (a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3), (c, 1), (c, 2), (c, 3) \}$$

Therefore,

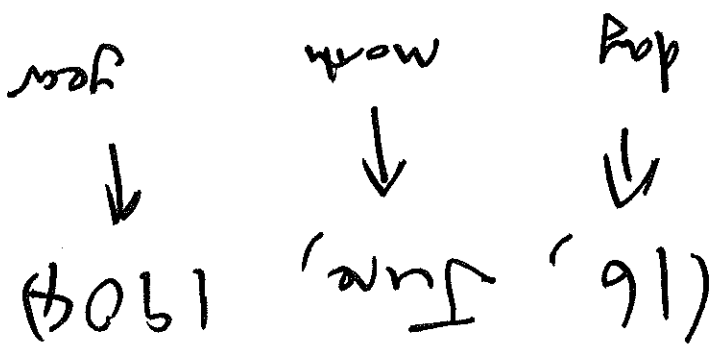
$$A \times B \neq B \times A$$

If $A = B$ we usually write A^2 instead of $A \times A$.

More generally, let A_1, \dots, A_n be sets

$$A_1 \times \dots \times A_n = \{ (a_1, \dots, a_n) : a_i \in A_i, \dots, a_n \in A_n \}$$

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Example Dates are really ordered tuples

(2) $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, real space.

(1) $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, the real plane.

Examples

$$A^n = \underbrace{A \times \dots \times A}_n \text{ times.}$$

If $A = A_1 = A_2 = \dots = A_n$ we write

$$\text{The probability of } X = \frac{36}{3} = \frac{12}{1}$$

$$X = \{(5,6), (6,6), (6,5)\}$$

Consider the following event

If $X \in \Omega$, we can call X an event.

The probability of event $X = \frac{|X|}{|\Omega|}$

Called the sample space.

$$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

possible outcomes in

Example If we throw two dice the set of

Example
 A menu in a restaurant gives you the following choices:

Starters: haggis, soup, prawns

Main course: steak, fish, vegetarian, vegan

Desserts: jelly, ice-cream.

Let $S = \{ \text{haggis, soup, prawns} \}$

$M = \{ \text{steak, fish, vegetarian, vegan} \}$

$D = \{ \text{jelly, ice-cream} \}$

~~A menu~~ The set of all possible meals is

therefore $S \times M \times D$. A particular

meal might be (haggis, steak, jelly)

order matters.

Example A square is a message

153 characters long. A character

is a part of or one of the 26 upper-case

Latin letters. Let

$$C = \{A_1, \dots, Z, \perp\}$$

↓
idle symbol.

A square is n characters in length. C

$$C^{153 \times 153}$$

$$|S \times M \times D| = |S| |M| |D| = 3 \times 4 \times 2 = \underline{\underline{24}}$$

Expe How many possible meals are there?

Expe How many equations are there?
 $|C| = |C| = \underline{\underline{27}}$

$$|X^2| = |X|^2 = 6^2 = \underline{\underline{36}}$$

Expe let $X = \{1, 2, 3, 4, 5, 6\}$

$$|A_1 \times \dots \times A_n| = |A_1| \dots |A_n|$$

Theorem 4

Let X be a set with $|X| = n$.

X^k consists of all k -tuples of

elements of X with k components. There are

$$|X|^k = n^k \text{ of them.}$$

Example $X = \{a, b, c\}$ or $k = 2$.

$$X^2 = \{ (a,a), (a,b), (a,c), (b,a), (b,b), (b,c),$$

$$(c,a), (c,b), (c,c) \}$$

$$|X^2| = 3^2 = \overline{9} \text{ elements.}$$