Lecture 4

Set notation does not enable us to take account of order and repetitions. To do this, we need the following notation.

round brackets

An <u>ordered pair</u> (a, b) has a <u>first component a</u> and a <u>second component b</u>. An <u>ordered triple</u> (a_1, a_2, a_3), an <u>ordered quadruple</u> (a_1, a_2, a_3, a_4) .... more generally, an <u>ordered n-tuple</u> $(a_1, \dots, a_n)$. The key feature of ordered pairs (and, more generally, ordered n-tuples) is that

$$(a, b) = (c, d) \text{ iff } a = c \text{ and } b = d$$

"if and only if"

$$(a_1, \dots, a_n) = (b_1, \dots, b_n) \text{ iff } a_i = b_i \text{ for all } 1 \leq i \leq n.$$

In general, $(a, b) \neq (b, a)$ order matters

(a, a) repetition allowed.

Let A and B be sets. Define the <u>product of A and B</u>, written $A \times B$, to be the set

$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}.$$

<u>Example</u> Let $A = \{1,2\}$ and $B = \{a,b,c\}$.

Then

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c)\}$$

$$B \times A = \{(a,1),(b,1),(c,1),(a,2),(b,2),(c,2)\}$$

Therefore, $A \times B \neq B \times A$.

If $A = B$ we usually write A^2 instead of $A \times A$.

More generally, let $A_1, \ldots, A_n$ be sets.

Then

$$A_1 \times \ldots \times A_n = \{(a_1, \ldots, a_n) : a_1 \in A_1, \ldots, a_n \in A_n\}$$

If $A = A_1 = A_2 = \dots = A_n$ we dfe

$$A^n = \underbrace{A \times \dots \times A}_{n \text{ times}}.$$

Examples

(1) $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$, te real plane.

(2) $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$, real space.

Example Dates are really ordered triple

$$(\underset{\substack{\uparrow \\ \text{day}}}{16}, \underset{\substack{\uparrow \\ \text{month}}}{\text{June}}, \underset{\substack{\uparrow \\ \text{year}}}{1904})$$

$1 \leq \text{day} \leq 31$

<u>Example</u> If we throw two dice the set of possible outcomes is

$$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

Called the <u>sample space</u>.

If $X \subseteq \Omega$, we call X an <u>event</u>.

The probability of event X $= \dfrac{|X|}{|\Omega|}$

Consider the following event

$$X = \{(5,6), (6,6), (6,5)\}$$

The probability of X $= \dfrac{3}{36} = \underline{\underline{\dfrac{1}{12}}}$

Example A menu in a restaurant gives you the following choices:

Starter: haggis, soup, prawns

mains: steak, fish, vegetarian, vegan

desserts: jelly, ice-cream.

Let S = {haggis, soup, prawns}

M = {steak, fish, vegetarian, vegan}

D = {jelly, ice-cream}

~~A meal~~ The set of all possible meals is therefore $S \times M \times D$. A particular meal might be $\underbrace{(\text{haggis, steak, jelly})}_{\text{order matters.}}$

<u>Example</u> A <u>squawk</u> is a message 153 characters long. A character is either a space or one of the 26 upper-case Latin letters. Let

$$C = \{A, \ldots, Z, \sqcup\}$$

$\uparrow$ space symbol.

A squawk is therefore an element of C^{153}

$$\Big(= \underbrace{C \times \cdots \times C}_{153 \text{ times}}\Big).$$

Theorem 4

$$\boxed{|A_1 \times \dots \times A_n| = |A_1| \dots |A_n|}$$

Example Let $X = \{1, 2, 3, 4, 5, 6\}$

$$|X^2| = |X|^2 = 6^2 = \underline{\underline{36}}.$$

Example How many equations are there?

$$|C^{153}| = |C|^{153} = \underline{\underline{27}}^{153}$$

Example How many possible meals are there?

$$|S \times M \times D| = |S||M||D| = 3 \times 4 \times 2 = \underline{\underline{24}}$$

Let X be a set with $|X| = n$.

X^k consists of <u>all</u> k-tuples of elements of X with k components. There are $|X|^k = n^k$ of them.

<u>Example</u> $X = \{a, b, c\}$ and $k = 2$.

$$X^2 = \{ (a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c) \} \;\&$$

$$|X^2| = 3^2 = \underline{\underline{9}} \text{ elements.}$$
