Lecture 5

Last time

An element of $A_1 \times \dots \times A_k$ is a k-tuple.

The number of k-tuples is $|A_1| \dots |A_k|$.

Now let $A = A_1 = \dots = A_k$

A k-tuple from A^k is called a k-permutation if it contains no repetitions.

If $|A| = n$. Then an n-permutation is called simply a permutation.

Example Let $X = \{a, b, c\}$, $k = 2$.

The following are all the 2-permutations of X:

$$\{(a,b), (a,c), (b,a), (b,c), (c,a), (c,b)\}$$

— 6 elements. Repetitions such as (a,a), (b,b), (c,c) have been weeded out.

<u>Theorem 5</u> Let X be a set with $|X| = n$ elements. Let $0 \leq k \leq n$. Then the number of <u>k-permutations</u> of X is

$$nP_k = \frac{n!}{(n-k)!}$$

<u>Example</u> $3P_2 = \frac{3!}{1!} = \underline{\underline{6}}$

Observation:

$$nP_0 = \frac{n!}{n!} = \underline{\underline{1}}$$

$$nP_n = \frac{n!}{0!} = \underline{\underline{n!}}$$

Example The permutations of $\{a, b, c\}$ are:

(a,b,c), (a,c,b), (b,a,c), (b,c,a), (c,a,b), (c,b,a)

$^3P_3 = 3! = \underline{6}$

Example How many permutations of the ~~set~~ elements of the set $\{a, b, c, d\}$ are there? $^4P_4 = 4! = 4.3.2 = \underline{24}$

(a, b, c, d) ..... (d, c, b, a)

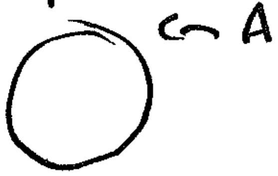
$|X| = n$

K-subsets Choosing no order, no repetitions	^nC_k
K-tuples Order + repetitions.	n^K
K-permutations order + no repetitions. ranking	^nP_k

Boolean operations on sets

The word 'Boolean' comes from the name Boole as in George Boole.

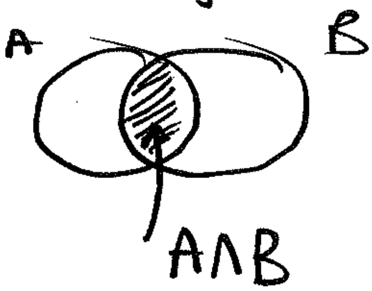
__Venn diagram__ Represent a set A by a region in the plane



__Definitions__ Let A and B be sets.

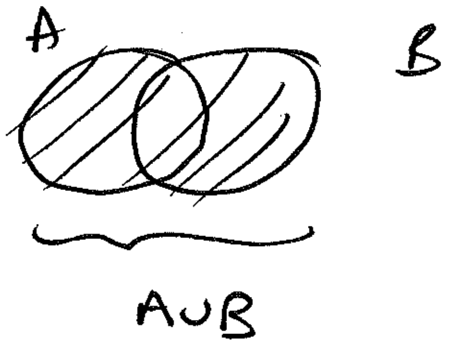
$$A \cap B = \{a : a \in A \text{ and } a \in B\}$$

called the __intersection of A and B__



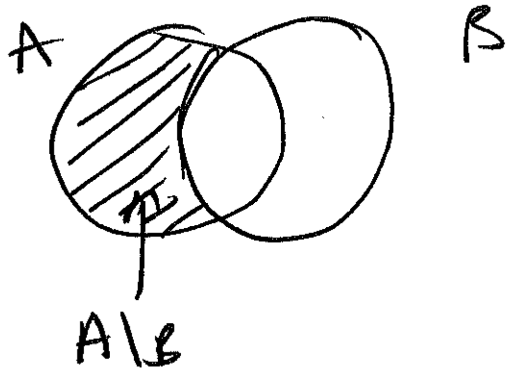
$$A \cup B = \{a : a \in A \text{ or } a \in B \text{ or both}\}.$$

called the __union of A and B__



$A \setminus B = \{a : a \in A \text{ and } a \notin B\}$

called the relative complement of A and B



Examples

Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \setminus B = \{1, 2\}$

$B \setminus A = \{5, 6\}$

If $A \cap B = \emptyset$ we say A and B are disjoint

Example How many elements are in $A \cup B$?

In general $|A \cup B| \neq |A| + |B|$

Let $A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

Then $A \cup B = \{1, 2, 3, 4\}$

$\therefore$ $|A \cup B| = 4$ but $|A| = 3 = |B|$

& $4 \neq 3 + 3$

The problem is that $A \cap B$ $(= \{2, 3\}) \neq \emptyset$

and so these elements are counted twice.

$\boxed{\text{If } A \cap B = \emptyset \text{ then } |A \cup B| = |A| + |B|}$

Definition Let X be a non-empty set. A **binary operation** $*$ on X is a function which associates with every ordered pair $(x, y) \in X^2$ an (= one) element $x * y \in X$.

Binary operations are common in Maths:

$\mathbb{N} \times \mathbb{N} \to \mathbb{N}$ given by $(x, y) \mapsto x+y$

$\mathbb{N} \times \mathbb{N} \to \mathbb{N}$ —— $(x,y) \mapsto xy$

are both binary operations.