Lecture 8.

2. Polynomials

This section has important applications in Calculus (in particular).

Section 4.3

Deriving the formula for the roots of a quadratic

Let $a, b, c \in \mathbb{R}$ where $a \neq 0$

Then $ax^2 + bx + c$ is called a quadratic polynomial or a polynomial of degree 2.

The equation

leading coefficient

$$\boxed{a}x^2 + bx + c = 0 \qquad (*)$$

is called a quadratic equation. If r is (for the time being) a real number s.t.

$$ar^2 + br + c = 0$$

then it is called a root of the quadratic equation.

$ax^2 + bx + c = 0$ where $a \neq 0$.

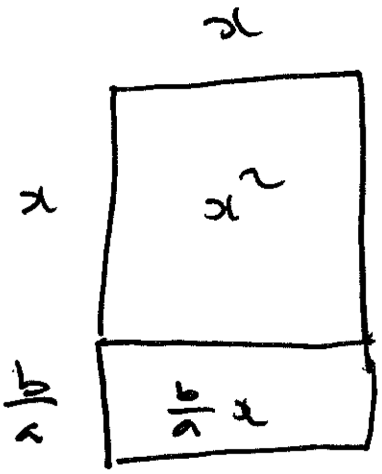
We may therefore divide both sides of this equation by a to get

monic polynomial Constant

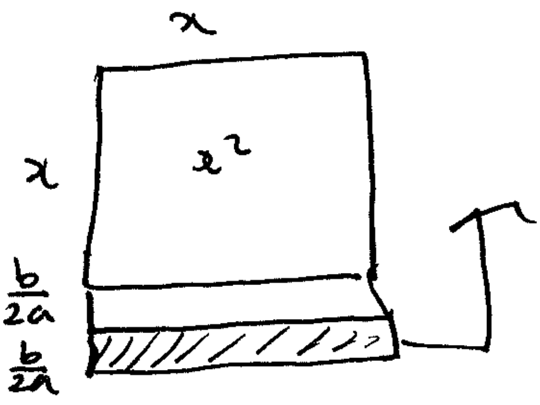
$$x^2 + \frac{b}{a}x + \boxed{\frac{c}{a}} = 0.$$

Focus on the part $\boxed{x^2 + \frac{b}{a}x}$ first.

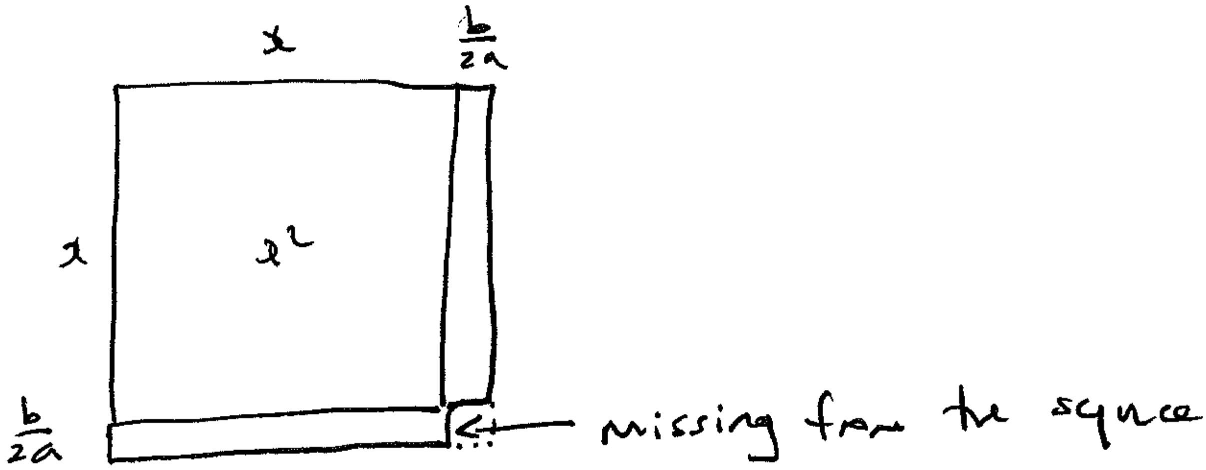
We draw a picture of $x^2 + \frac{b}{a}x$:



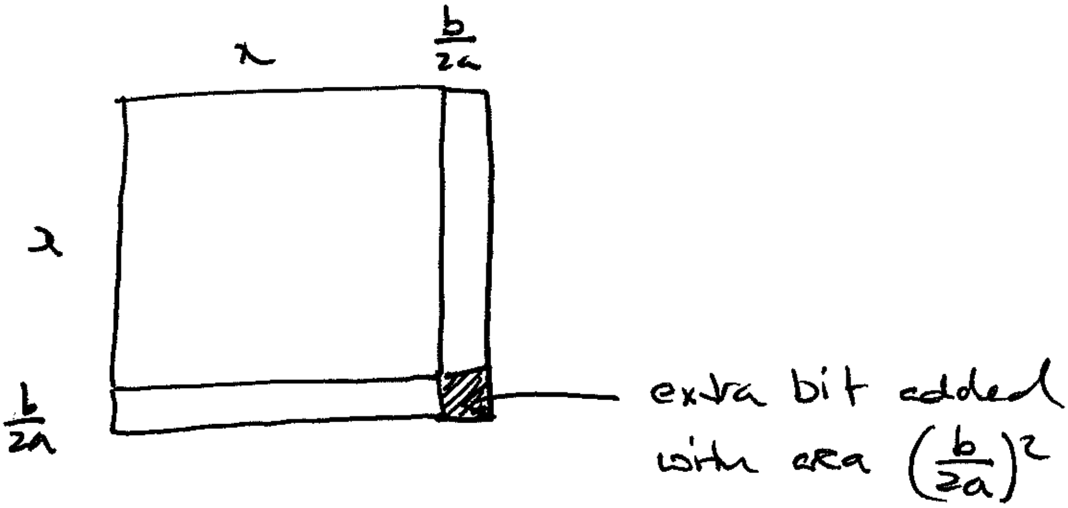
cut this in half horizontally



move this part to get...



Complete the square



Area of completed square is $\left(x + \frac{b}{2a}\right)^2$

$$\boxed{x^2 + \frac{b}{a}x = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2}$$

This algebraic identity can be checked directly
but the pictures explain why this process is called <u>Completing the square</u>

$$\boxed{x^2 + \frac{b}{a}x} + \frac{c}{a} = 0$$

replace by

$$\boxed{\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2} + \frac{c}{a} = 0$$

This can be rewritten as

$$\left(x + \frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$= \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$\therefore \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

Thus

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Define $D = b^2-4ac$.

This is called the <u>discriminant</u> of our quadratic.

$$\therefore \quad x = \frac{-b \pm \sqrt{D}}{2a}.$$

(1) If $D > 0$ then our quadratic equation has two distinct real roots

(2) If $D = 0$ then our quadratic equation has a repeated root (what this means will be explained later)

(3) If $D < 0$ we say that our polynomial is ~~real~~ a real irreducible quadratic. This case will be very important to us later..