F17CC SPECIMEN EXAM PAPER

Each question part is worth 5 marks. The whole paper is worth 80 marks.

1. (a) How many subsets does the set $\{a, b, c, d\}$ have?

   (b) Write the complex number $\frac{2+3i}{4+i}$ in the form $a + bi$ where a and b are real numbers.

   (c) Carry out the following matrix multiplication

   $$\begin{pmatrix} 0 & 4 & 2 \\ -1 & 1 & 3 \\ 2 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 2 & 0 & -4 \\ 3 & 2 & 0 \end{pmatrix}$$

   (d) Find the angle to the nearest degree between the vectors $\mathbf{a}$ and $\mathbf{b}$ where

   $$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}.$$

2. (a) In how many ways can a hand of 13 cards be chosen from a pack of 52 cards?

   (b) Find the square roots of $-1 + i\sqrt{24}$ and show that your solutions work.

   (c) Solve the following system of linear equations. You **must** use elementary row operations. Show that your solution(s) work.

   $$\begin{aligned} 4x + 4y - 8z &= 20 \\ 3x + 2y + 2z &= 1 \\ 5x + 4y + 3z &= 4. \end{aligned}$$

   (d) Calculate the vector product $\mathbf{a} \times \mathbf{b}$ of the vectors $\mathbf{a}$ and $\mathbf{b}$ where

   $$\mathbf{a} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}.$$

Exam paper continues . . .

3. (a) Let $A = \{a, b, c, d, e, f\}$ and $B = \{g, h, k, d, e, f\}$. What are the elements of the set

$$A \setminus ((A \cup B) \setminus (A \cap B))?$$

(b) Factorize $x^4 + 1$ as a product of real linear and real irreducible quadratic polynomials.

(c) Use De Moivre's theorem to express $\sin 5x$ in terms of $\cos x$ and $\sin x$.

(d) Find the determinant of the matrix below.

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}.$$

4. (a) Solve the following system of linear equations. You **must** use elementary row operations. Show that your solutions work.

$$\begin{aligned} 5x + 4y - 2z &= 3 \\ 3x + 3y - z &= 2 \\ 2x + 4y + 0z &= 2. \end{aligned}$$

(b) Find the inverse of the matrix below **by using the adjugate**, and show that your answer works

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ -1 & 1 & 2 \end{pmatrix}.$$

(c) The plane $px + qy + rz = s$ contains the point with position vector $\mathbf{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and has normal $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. Find p, q, r, s.

(d) Find the characteristic polynomial and eigenvalues of the matrix below.

$$\begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{pmatrix}.$$

End of paper