Introduction to University Mathematics
Test 1 Friday 11th October 10.20 to 11.10

NAME: (please PRINT)

Circle one of the following:

MATHS   AMS   OTHER

This test is worth 10% of your final grade. It will be marked and
handed back during the tutorials next week. It is a closed book test.
Full answers should be written in the spaces provided. University rules
about cheating apply. The test is designed to last no more than 15
minutes but you can stay the whole 50 minutes if you wish. **There
are 5 questions, each worth 2 marks.**

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1. A licence plate consists of 3 upper case letters followed by 4 digits
(a *digit* is an element of the set $\{0,1,2,3,4,5,6,7,8,9\}$). How many
such licence plates are there?

Put $\mathcal{A} = \{A, B, ..., Z\}$ and $\mathcal{D} = \{0, 1, ..., 9\}$. A licence
plate is an element of the set $\mathcal{A}^3 \times \mathcal{D}^4$. But

$$|\mathcal{A}^3 \times \mathcal{D}^4| = |\mathcal{A}|^3 |\mathcal{D}|^4 = 26^3 \cdot 10^4 = 175,760,000$$

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2. A coin is tossed 10 times. How many outcomes are there?

Let H = heads and T = tails.

The set of all possible outcomes is $\{H, T\}^{10}$.

The number of outcomes is

$$|\{H, T\}^{10}| = 2^{10} = 1024$$

3. A coin is tossed 10 times. How many times can exactly 3 heads occur?

Choose 3 coin tosses out of 10 to be heads.

This is $\binom{10}{3} = 120.$

4. 10 politicians fall into a vat of fondue one after the other. In how many ways can this occur?

Order matters, no repetition: this is a permutation. The number of ways is therefore

$$10! = 3,628,800$$

5. Let $A = \{a, b, c, d, e\}$, $B = \{d, e, f\}$ and $C = \{1, 2, 3\}$. Write down the elements of the set

$$(A \times C) \cap (B \times C).$$

$$\{d, e\} \times \{1, 2, 3\}$$