Test 1 F17CC Friday 5th October: solutions

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1. A dice, a coin and a tetrahedron are thrown at the same time. How many possible outcomes are there?

It goes without saying that you should all know what a tetrahedron is; you lose a mark if you use the right method but don't know that a tetrahedron has four sides. Although not necessary for what follows, imagine the faces of the tetrahedron labelled red, white, blue and green. The set of possible outcomes is therefore the product set

$$\{1,2,3,4,5,6\} \times \{\textit{red, white, blue, green}\} \times \{H,T\}.$$

The number of possible outcomes is the cardinality of this set which is $48 = 6 \times 4 \times 2$*. This follows by the third counting principle described in Lecture 5.* **Questions 1(a),(b),(j) of Exercises 3 are all of this type.**

2. Let $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$ and $C = \{1,2,7,8\}$. Write down the elements of the following set (to get the marks, your notation must be exact):

$$[(A \cap B) \times (A \cap C)] \setminus \{(3,1)\}.$$

The operations needed to answer this question were described in Lecture 3. Observe that

$$A \cap B = \{3,4\} \textit{ and } A \cap C = \{1,2\}.$$

It follows that

$$(A \cap B) \times (A \cap C) = \{(3,1),(3,2),(4,1),(4,2)\}.$$

Thus

$$[(A \cap B) \times (A \cap C)] \setminus \{(3,1)\} = \{(3,2),(4,1),(4,2)\}.$$

Quite a few students are simply ignoring the symbol $\times$ *completely. Clearly, if you ignore notation, you are not going to get the right answer.* **Questions 7, 8 and 9 of Exercises 2 are all of this type.**

3. A group of 30 politicians runs up Arthur's Seat. Assuming no ties, how many outcomes are there for first, second and third position in this competition?

This is a question about counting k-perms. Allowable solutions are $30.29.28$ *or* $24,360$ *or* $\frac{30!}{27!}$*. I really do wonder about those of you who* *wrote* $\frac{30!}{(30-3)!}$. *Have you taken an oath against doing elementary arithmetic?* **Questions 1(c), (d), (e), (h) of Exercises 3 are all of this type.**

4. A *squawk* is a message 42 characters long where a character is either one of the 26 letters of the Latin alphabet or a space symbol. How many squawks are there?

This is a question about counting k-tuples. Although not necessary for what follows, write $\square$ for the space symbol. A squawk is therefore an element of $\{A, B, \ldots, X, Y, Z, \square\}^{42}$. By the third counting principle, the cardinality of this set is 27^{42}. Some of you are keen to use your calculators and work out this number explicitly. However, (a) what your calculator works out is not an exact answer but an approximation and (b) you don't get marks for knowing how to use a pocket calculator. The issue of getting numbers out in probability theory that you can trust is actually a delicate one and cannot be solved by simply using a calculator. **Question 2 of Exercises 3 is of this type**. *One or two of you were worried about the spaces. You should not have been. The question tells you what a squawk is. And yes, a squawk might just be 42 blank spaces — the best kind, if you ask me.*

5. How many 3 person committees are there that can be formed from 20 university administrators?

This is a question about counting k-subsets. Allowable solutions are $\binom{20}{3}$ *or* $1,140$ *or* $\frac{20!}{3!17!}$. **Questions 1(f), (g), (i) of Exercises 3 are all of this type.**

6. **Bonus question.** *This will not contribute to your grade for this test.* Find all $x, y, z \in \mathbb{N}$ such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1.$$

Observe that $x, y, z \geq 2$. Assume, without loss of generality, that $x \leq y \leq z$. The key step is to observe that when $x \geq 5$, we have that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \leq \frac{3}{5},$$

which means that we cannot get the lefthand side to equal 1. It follows that we need only check for solutions when $x = 2, 3, 4$. The case where

$x = 2$. Then we need

$$\frac{1}{y} + \frac{1}{z} = \frac{1}{2}$$

where $2 \le y \le z$; $y = 2$ is impossible; $y = 3$ gives $z = 6$; $y = 4$ gives $z = 4$; $y \ge 5$ is impossible. The case where $x = 3$. Then we need

$$\frac{1}{y} + \frac{1}{z} = \frac{2}{3}$$

where $3 \le y \le z$; $y = 3$ gives $z = 3$; $y \ge 4$ is impossible. The case where $x = 4$ is impossible. Thus the solutions (x, y, z), where $x \le y \le z$, are $(2, 3, 6), (2, 4, 4), (3, 3, 3)$.