Test 2 F17CC Friday 2nd November 10.20 to 11.10

NAME: (please PRINT)

Circle one of the following:

MATHS AMS OTHER

This test is worth 10% of your final grade. It will be marked and handed back during the tutorials next week. It is a closed book test. Full answers should be written in the spaces provided. University rules about cheating apply. **There are 5 questions, each worth 2 marks.**

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1. Write

$$\frac{82+123i}{4+5i}$$

in the form $a+bi$ where a and b are rational.

The method to solve this question was described in Lecture 10:

$$\frac{82+123i}{4+5i}=\frac{(82+123i)(4-5i)}{(4+5i)(4-5i)}=23+2i.$$

[1 mark] for method and [1 mark] for getting the right answer. **Question 1 of Exercises 6 is of this type.**

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2. Find all the square roots of $84i-13$. Your solutions MUST be written in the form $a+bi$ where a and b are real. You MUST check your solutions.

The method needed to answer this question was described in Lecture 11. **Question 2 of Exercises 6 is of this type.** *We need to find* $(x+iy)^2 = 84i-13$ *(*). This leads to the two equations* $x^2-y^2=-13$ *and* $2xy=84$*. We obtain the third equation by taking the moduli of both sides of (*)* $x^2+y^2=85$*. The roots are* $\pm(6+7i)$*. You check a root by squaring it — this proves that your answer is correct.*

3. Find all fourth roots of 16.

The method needed to answer this question was described in Lectures 14 and 15. There are two ways to answer this question. The first is to remember that the fourth roots of unity are ± 1 *and* $\pm i$*. A specific 4th root of 16 is 2 (since* $2^4 = 16$*). Thus the 4th roots of 16 are* $2, -2, 2i, -2i$*. The second method is trigonometric. However, it should lead you to the same answers since in this case the trig solutions can easily be written in radical form. If you don't give any complex roots you get [0 marks]. Otherwise, you get [*$\frac{1}{2}$ *mark] for each correct root.* **Questions 10, 11, 12, 13, 14 of Exercises 7 are of this type**.

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4. Write $x^5 + 4x^4 + 5x^3 - 4x^2 - 42x - 36$ as a product of real linear and real irreducible quadratic polynomials.

The method needed to solve this question was described in Lecture 16. The given polynomial is monic with integer coefficients. We therefore know the integral roots will be divisors of -36*. After some effort, you find that* $-1, 2, -3$ *all work. Dividing out by the product of the corresponding linear factors we get an irreducible quadratic. The required factorization is therefore*

$$(x+1)(x-2)(x+3)(x^2+2x+6).$$

Questions 2, 6, 7, 8, 9 of Exercises 7 are of this type.

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5. Find the coefficient of the term in $x^{24}y^6$ in the binomial expansion of $(-2x+3y)^{30}$.

The method needed to solve this question was described in Lecture 9. By the binomial theorem

$$(-2x+3y)^{30} = ((-2x)+(3y))^{30} = \sum_{i=0}^{30} \binom{30}{i} (-2x)^i (3y)^{30-i} = \sum_{i=0}^{30} \binom{30}{i} (-2)^i 3^{30-i} x^i y^{30-i}.$$

The summand when $i = 24$ *is therefore* $\binom{30}{24}(-2)^{24}3^6x^{24}y^6$*. The coefficient — which is a number — is therefore* $\binom{30}{24}(-2)^{24}3^6 = \binom{30}{24}2^{24}3^6$*. Some of you* *like to play with your calculators to work out this number — but you only get an approximate answer (so why do it?). Some of you think that brackets are mere decoration (they are not).* **Exercises 5 is dedicated to the binomial theorem**.

Some lessons ...
Observe that:

1. All the methods needed to answer these questions were described in the lectures.

2. The exercises contained questions of each of the above types.

3. The exam will be more the same. It is an example of 'magical thinking' if you believe that you can pass the exam without reading your notes or doing the exercises.