Introduction to University Mathematics
Test 2 Friday 8th November 10.20 to 11.10

NAME: (please PRINT)

Circle one of the following:

MATHS AMS OTHER

This test is worth 10% of your final grade. It will be marked and handed back during the tutorials next week. It is a closed book test. Full answers should be written in the spaces provided. University rules about cheating apply. The test is designed to last no more than 15 minutes but you can stay the whole 50 minutes if you wish. **There are 2 questions, each worth 5 marks.**

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1. Find both square roots of the complex number

$$-45 + 28i$$

and show that your solutions work.

Let $(x+iy)^2 = -45 + 28i$.

Then ① $x^2 - y^2 = -45$

② $2xy = 28$

③ $x^2 + y^2 = \sqrt{45^2 + 28^2}$

$= 53$

① + ③ gives $2x^2 = 8$. Thus $x^2 = 4$.

It follows that $x = \pm 2$.

By ② $y = \frac{28}{2x} = \frac{14}{x}$

Thus when $x = 2$, $y = 7$ and when $x = -2$, $y = -7$.

The square roots of $-45 + 28i$ are therefore $\pm(2+7i)$

[4 marks]

Check $(2+7i)^2 = 4 + 28i - 49$

$= -45 + 28i$ ✓ [1 mark]

* Complex numbers are written $a+ib$ not as ordered pairs

* You check your solutions by squaring them not by multiplying them together

2. Write the following real polynomial as a product of real linear and real irreducible quadratic polynomials

$$x^4 - 7x^3 + 18x^2 - 22x + 12.$$

You should make clear why any real quadratics that appear are irreducible.

Let $p(x) = x^4 - 7x^3 + 18x^2 - 22x + 12$.

Any integer roots of $p(x)$ must divide 12.

Try $\pm 1, \pm 2, \pm 3, \ldots$

We find that 2 and 3 work (ie $p(2) = 0$, $p(3) = 0$).

Divide $p(x)$ by $(x-2)(x-3)$ to get $x^2 - 2x + 2$.

[4 marks]

Thus

$$p(x) = (x-2)(x-3)(x^2 - 2x + 2)$$

The discriminant of $x^2 - 2x + 2$ is $(-2)^2 - 4 \cdot 2 = -4 < 0$

It follows that $x^2 - 2x + 2$ is irreducible

→ [1 mark] need discriminant of $x^2 - 2x + 2$ is negative.

The discriminant of $ax^2 + bx + c$ is $b^2 - 4ac$

It is a real number (not in $\mathbb{C}\setminus\mathbb{R}$)