Lecture 1

Introduction to university Mathematics F17CC

This course is a bridge between school and university mathematics (a bridge takes you from where you are to somewhere else — otherwise it is a poor bridge*).

Please read and mull over Chapters 1 and 2 of my book — available free of charge on VISION. If you cannot see this course on your VISION page then please email me).

(*) A pier?

This course is in four parts:

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1. Combinatorics.

This is about counting. Counting may not sound very interesting but it is the basis of probability theory and statistics which are interesting.

[Physicists: you need stats in interpreting experiments, and in thermodynamics. You need probability theory in QM (quantum mechanics).

To count well requires a good notation to express our ideas. The notation we introduce is set theory. This will also be useful for the whole course.

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2. Algebra, complex numbers and polynomials.

This part of the course is mainly about understanding the roots of a polynomial.

A ~~—~~ polynomial is an expression that looks like this

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

where $a_0, a_1, \dots, a_n$ are numbers. A root of $f(x)$ is a number a such that (s.t.) $f(a) = 0$. Finding roots of polynomials is useful in almost all parts of Maths & physics

What do we mean by "number"?

It turns out ~~—~~ that the common-or-garden idea of numbers is not enough and needs to be extended to a new class of numbers called complex numbers.

Some of you might think: I am a practical down-to-earth sort of person and do not need such fancy new numbers.

Au contraire! There are calculations in stats, for example, where complex numbers are needed. QM depends on them.

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3. Matrices

These arose as "packaging" for linear equations and then developed a life of their own. Essential in all areas of Maths & physics.

Linear equations are the most basic types of equations there are. It is essential to be able to solve them efficiently (so that you can concentrate on more interesting questions).

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4. Vectors. We live in a 3D world but visualizing in 3D is hard. The theory of vectors is an algebraic tool to help us do just that. There is a very close connection between vectors and matrices (together they form a subject called linear algebra which is a basic element in any mathematical understanding even though it is non-linearity that we frequently have to deal with).

1. Combinatorics

The art of counting.

What do we count?

We count the number of <u>elements</u> in a <u>set</u>.

We start, therefore, with some basic set theory.

<u>Basic set theory</u>

See Section 3.1 of my book for more information.

<u>"Definition"</u> A *set* is a collection of things, called its *elements*, which we wish to regard as a whole.

This is not really a good definition in a mathematical sense, instead it is intended to convey intuition about what a set is supposed to be.

"Definition" Two sets are equal when they contain exactly the same elements.

Example Here are some random things

$\square$, $\bigcirc$, $*$, $\uparrow$.

I wish to regard them as a single thing so

I collect them together in a set

$$\{\square, \bigcirc, *, \uparrow\}$$

"curly brackets" = braces tell you where the set begins and ends and commas separate the elements.

I can give this set a name (Fred, Daisy, ... but usually upper-case Latin ~~...~~ letters)

$$A = \{ \square, \bigcirc, *, \uparrow \}.$$

This defines what I am calling the set A.

$\square$ is <u>an element</u> of A we usually write this as $\square \in A$

$\in$ means 'is an element of'
it is like a shorthand symbol

So, $* \in A$.

BUT $\triangle \notin A$

$\notin$ means 'is not an element of'

Two important features of set notation:

— order doesn't matter. So,

$\{a,b,c\} = \{b,c,a\}$ etc.

— repetitions are ignored. So,

$\{a,a,b,b,c,c,c\} = \{a,b,c\}$

[These may seem odd but set notation works.]

Example $\{\ \}$ is the empty set. To avoid any confusion we give this set a special name

$\emptyset = \{\ \}$.