Lecture 3

Set Constructions

round brackets

~~Def~~ <u>Definition</u> An <u>ordered pair</u> (a, b) has a <u>first component</u> a and a <u>second component</u> b.

The key feature is that

$$(a, b) = (c, d) \text{ iff } a = c \text{ and } b = d.$$

<u>Definition</u> Let A and B be sets. Then

$$A \times B = \{ (a, b) : a \in A \text{ and } b \in B \}$$

is called the <u>product of</u> A <u>and</u> B

<u>Example</u> Let $A = \{1, 2\}$ and $B = \{a, b, c\}$.

Then $A \times B = \{ (1,a), (1,b), (1,c), (2,a), (2,b), (2,c) \}$

$B \times A = \{ (a,1), (b,1), (c,1), (a,2), (b,2), (c,2) \}$

$\boxed{\text{So, } A \times B \neq B \times A}$

If $A = B$ we write $A^2 = A \times A$.

<u>Definition</u> We can also define <u>ordered triples</u> (x, y, z) and more generally <u>ordered n-tuples</u> $(x_1, \ldots, x_n)$.

<u>Definition</u> Let $A_1, \ldots, A_n$ be sets.

$$A_1 \times \ldots \times A_n = \{ (a_1, \ldots, a_n) : a_1 \in A_1, \ldots, a_n \in A_n \}$$

If $A_1 = \ldots = A_n$ define

$$A^n = \underbrace{A \times \ldots \times A}_{n \text{ times}}.$$

Example

$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ the real plane

$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ real space.

Example Dates are really ordered triples

(16, June, 1904) (Bloomsday)

↑ day ↑ month ↑ year

Example Probability uses the language of set theory. Throw two dice. The set of possible outcomes is

$$\Omega = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$$

the sample space. Subsets of Ω are called events

If $X \subseteq \Omega$ the probability X is $\frac{|X|}{|\Omega|}$

$|\Omega| = 36$

Let $X \subseteq \Omega$ be the outcome where both dice show an even number

$X = \{2,4,6\} \times \{2,4,6\}$

$|X| = 9$

$\therefore$ the probability of two even is $\frac{9}{36} = \frac{1}{4}$

Definition Let A be a non-empty set. A **string over A** is an ordered sequence of elements of A. Let $x = a_1 \ldots a_m$ be a string over A. We say it has **length m**.

Example Let $A = \{0,1\}$. Strings over A are called **binary strings**.

0101 is a binary string of length 4

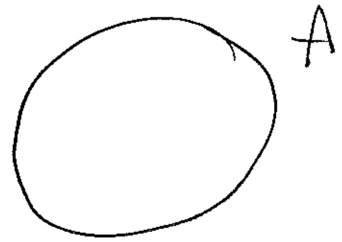
Example Let $A = \{x, y\}$. ~~strings~~

Strings over A look like x, y, xy, yx, xx, yy, xyx, etc.

Not discussed in lecture?
— will be discussed later

Boolean operations

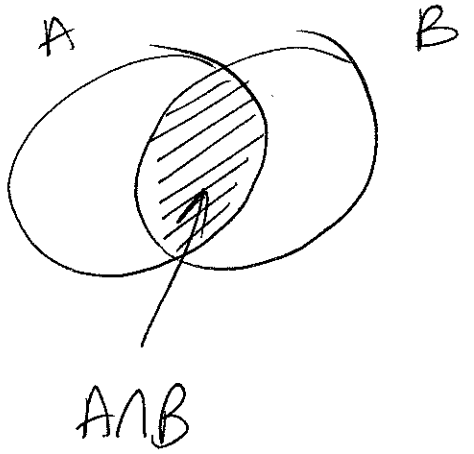
Venn diagrams. ~~Think of A~~ Represent A by a region in the plane



Definition Let A, B be sets.

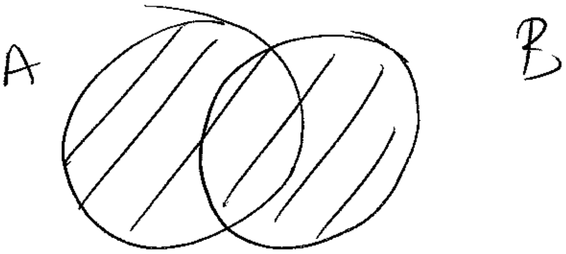
$$A \cap B = \{a : a \in A \text{ and } a \in B\}$$

called the *intersection* of A and B



$A \cup B = \{a : a \in A \text{ or } a \in B \text{ or both}\}$

Called the <u>union of A</u> <u>and</u> B



$A \setminus B = \{a : a \in A \text{ and } a \notin B\}$

Called the relative <u>complement of A and B</u>

Examples

(4) Let $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$

$A \cap B = \{3,4\}$

$A \cup B = \{1,2,3,4,5,6\}$

$A \setminus B = \{1,2\}$, $B \setminus A = \{5,6\}$

Definition sets A and B are said to be **disjoint** if $A \cap B = \emptyset$.

[Not discussed in lecture]