

# Lecture 18

## Truth trees Continued

Rem Truth trees are not proof trees

### $\alpha$ -rules

$$\begin{array}{c} X \wedge Y \\ | \\ X \\ | \\ Y \end{array}$$

$$\begin{array}{c} \neg(X \vee Y) \\ | \\ \neg X \\ | \\ \neg Y \end{array}$$

$$\begin{array}{c} \neg(X \rightarrow Y) \\ | \\ X \\ | \\ \neg Y \end{array}$$

$$\begin{array}{c} \neg\neg X \\ | \\ X \end{array}$$

### $\beta$ -rules

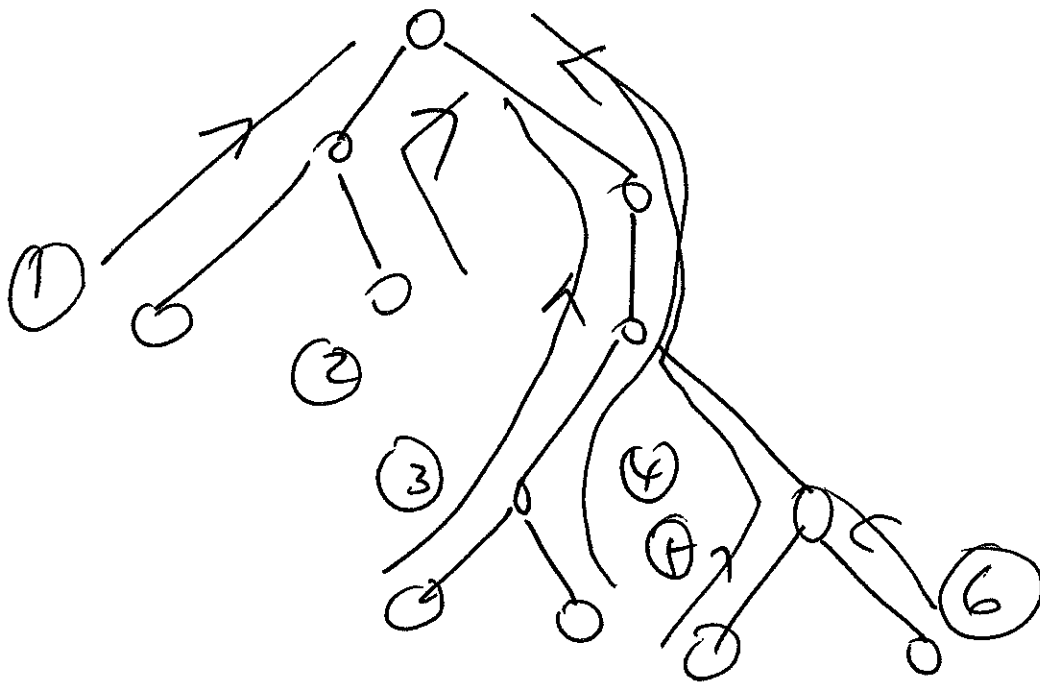
$$\begin{array}{c} X \vee Y \\ / \quad \backslash \\ X \quad Y \end{array}$$

$$\begin{array}{c} \neg(X \wedge Y) \\ / \quad \backslash \\ \neg X \quad \neg Y \end{array}$$

$$\begin{array}{c} X \rightarrow \neg \\ / \quad \backslash \\ \neg X \quad Y \end{array}$$

$$\begin{array}{c} X \leftrightarrow Y \\ / \quad \backslash \\ X \quad \neg X \\ \backslash \quad / \\ Y \quad \neg Y \end{array}$$

$$\begin{array}{c} \neg(X \leftrightarrow Y) \\ / \quad \backslash \\ X \quad \neg X \\ \backslash \quad / \\ \neg Y \quad Y \end{array}$$

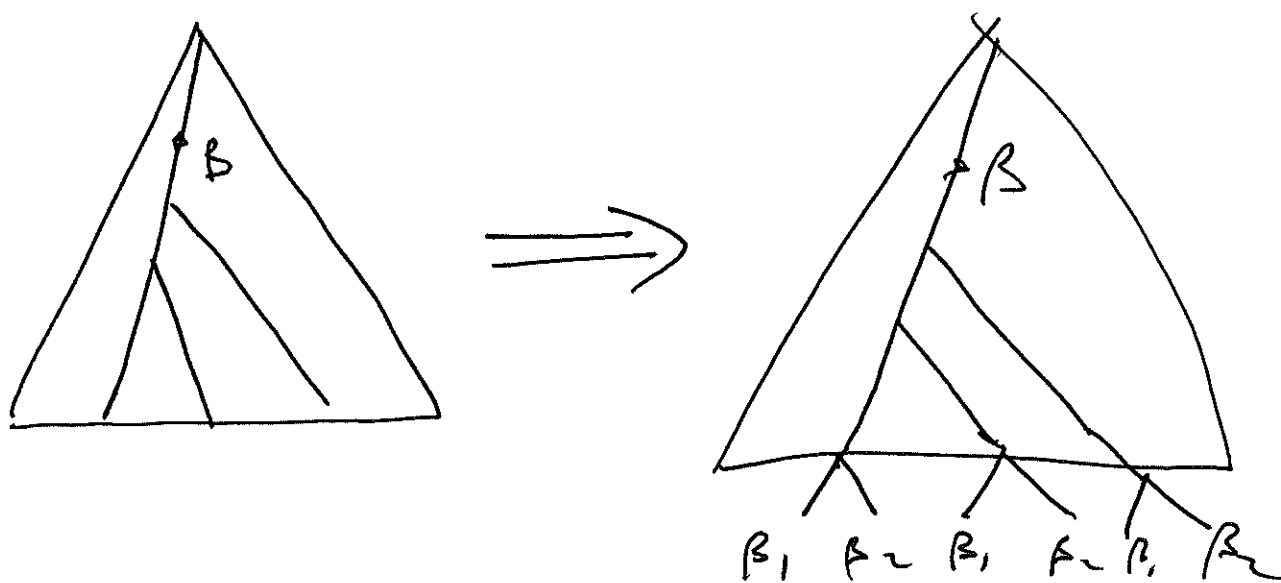
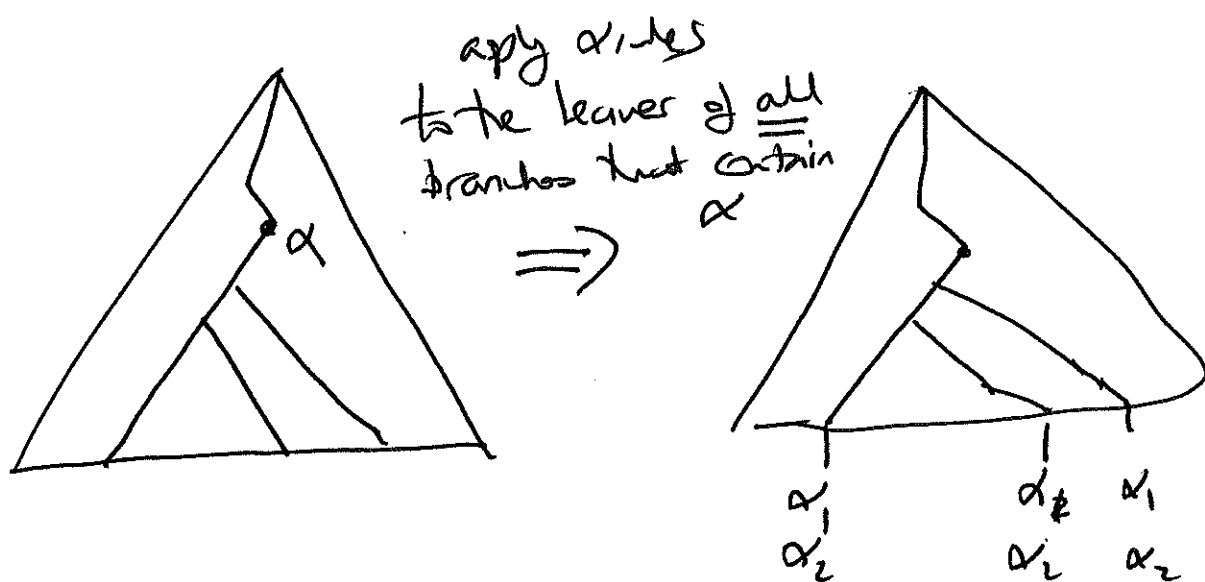


I have drawn the six branches in this tree  
 — a branch is a path from a leaf to  
 to root.

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1. Truth flows down branches.
2. Apply  $\alpha$ -rules before  $\beta$ -rules  
 (this is a strategy to improve efficiency  
 rather than anything to do with the  
 correct application of trees).

3. It is important to spy  $\alpha$ -nodes of  $\beta$ -nodes  $\Rightarrow$  follows:



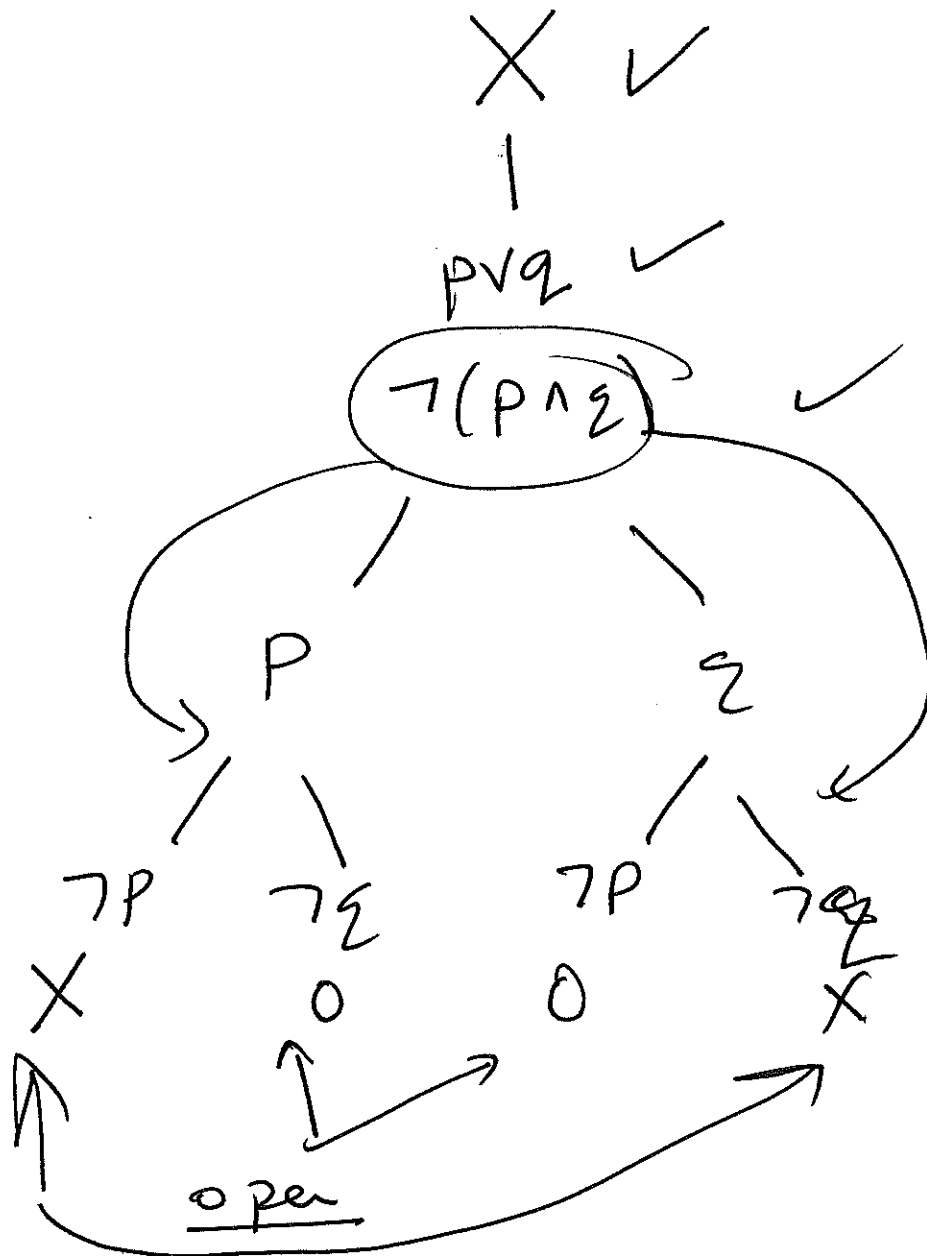

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4. If a branch contains  $\alpha$  and  $\neg\alpha$  then close at current leaf.

Example

Construct a truth tree for

$$X = \neg [(P \vee Q) \rightarrow (P \wedge Q)]$$

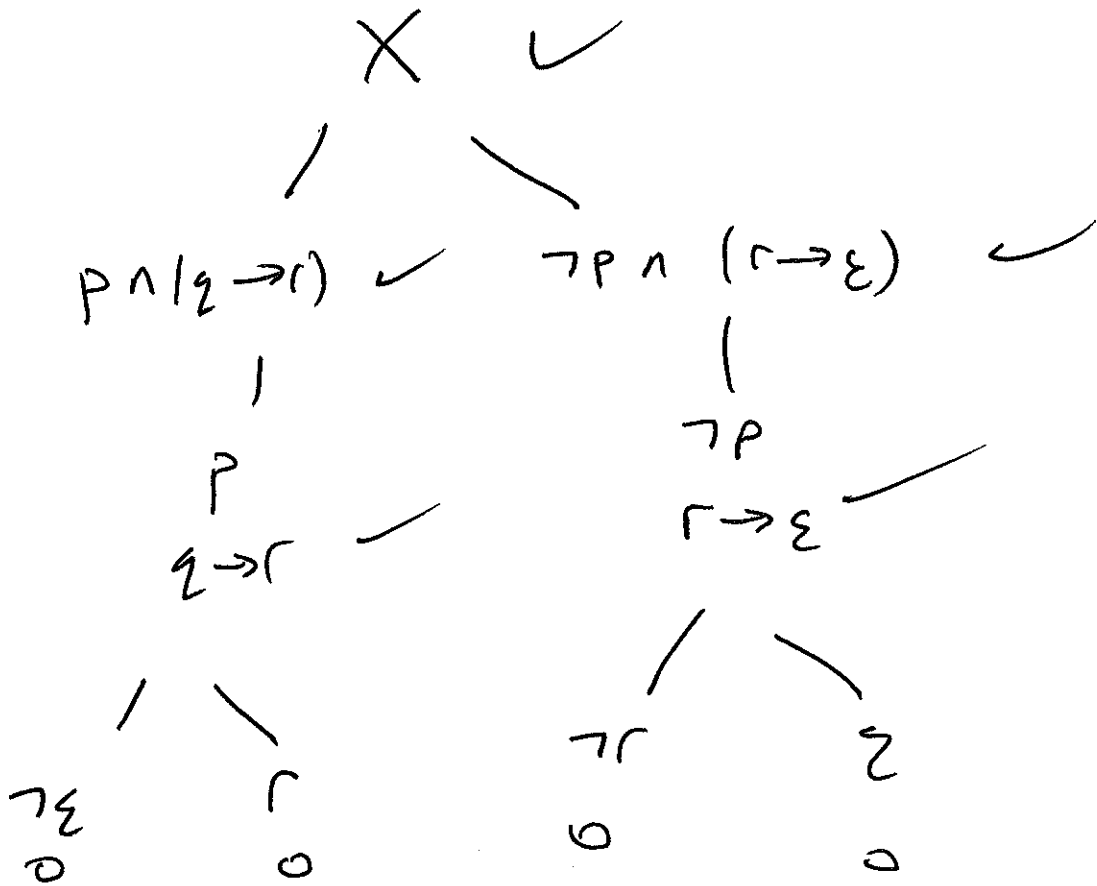


Closed

$$X \equiv (P \wedge \neg Q) \vee (\neg P \wedge Q) \quad \text{DNF}$$

Example Construct truth tree for

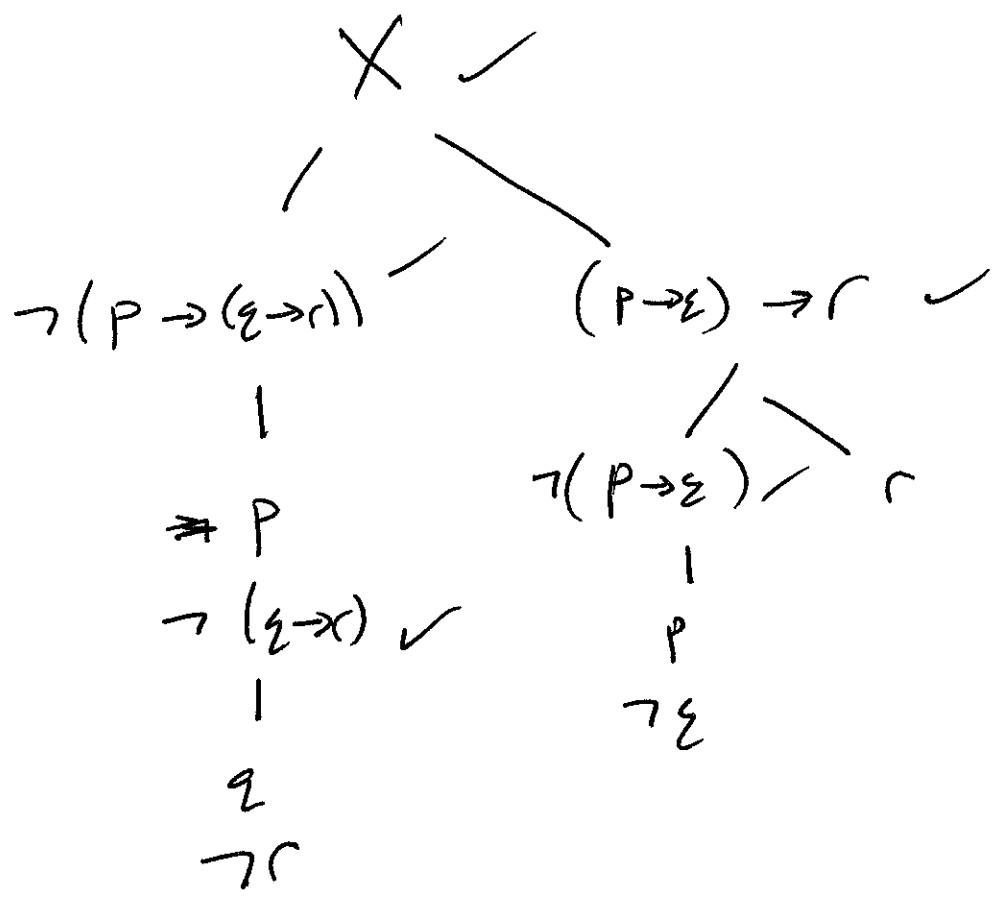
$$X = (P \wedge (Q \rightarrow R)) \vee (\neg P \wedge (R \rightarrow \Sigma))$$



$$X \equiv (\neg Q \wedge P) \vee (R \wedge P) \vee (\neg R \wedge \neg P) \vee (\Sigma \wedge \neg P)$$

Example Construct the truth tree of

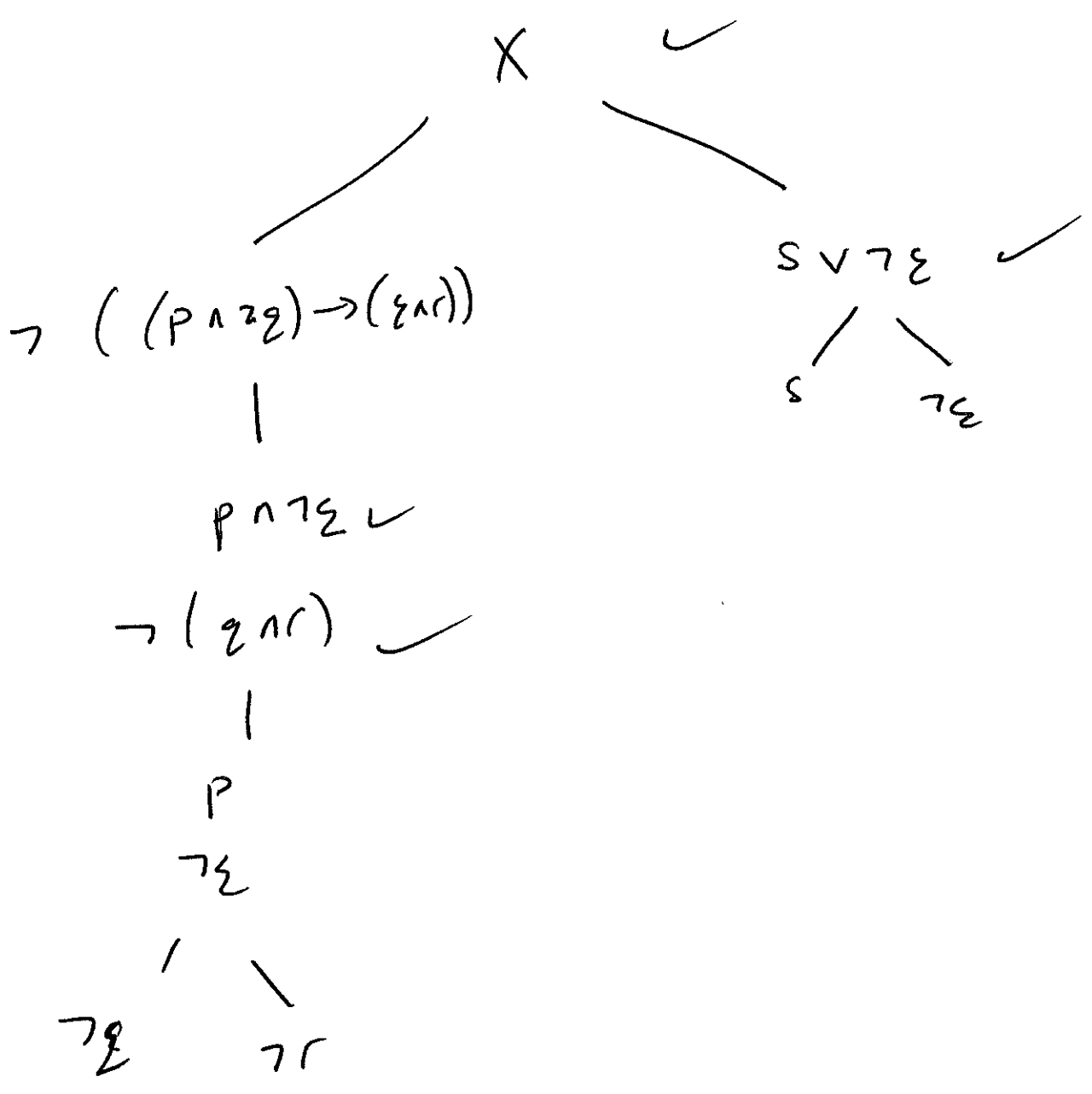
$$X = (P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow R)$$



$$X \equiv (P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q) \vee (R)$$

Example Construct a truth tree of

$$X = ((P \wedge \neg Q) \rightarrow (R \wedge \neg R)) \rightarrow (S \vee \neg S)$$



$$X \equiv (P \wedge \neg Q) \vee (P \wedge \neg Q \wedge \neg R) \vee (S \vee \neg S)$$

- If all branches of a truth tree close the wff at the root is a contradiction

- If some of the branches are open, the wff can be read to construct a wff in DNF which is logically equivalent to the wff at the root.