

Lecture 2

1. Propositional logic (PL)

Chapter 1, Section 1.1. Informal PL

PL is the logic of elementary decision making.

What could this mean?

Here is an example from my car's
handbook (written in American English).

Example

The door ajar warning lamp

illuminates

if the ignition is on

and either the door is open

or the hood is open.

We have to interpret this sentence.

The word 'if' here means

'exactly when' — the door ajar warning lamp illuminates precisely when the given conditions hold (and not otherwise)

To save time writing, I shall introduce some abbreviations:

$Q =$ 'The ignition is on'

$r =$ 'The door is open'

$S =$ 'The hood is open'

$A =$ 'The door ajar warning light illuminates'.

Each Q , r , or S is either true (T) or false (F).

The exact condition for A to be T is that Q and (r or S) is T (otherwise it is F).

I have written and or or because these words will be used with very special meanings (to avoid ambiguity).

- The English word 'and' is often used to mean 'and then' - We do not use this meaning here.

- The English word 'or' can be used with an exclusive meaning (p or q but not both) or an inclusive meaning (p or q or both). We only use it with an inclusive meaning.

We shall now draw up a table,
 called a truth table, that
 considers all the possible truth values
 of P , Q , or \neg and the
 corresponding truth value of A

[A is false means 'The door
 ajar lamp doesn't illuminate']

Each of P , Q , or \neg takes
 two values \therefore there are $2 \times 2 \times 2$
 $= 2^3$ possible truth assignments.

BE SYSTEMATIC!

one at a time

Two at a time

Four at a time

Q	R	S	$(r \text{ or } s)$	$\neg A$ $Q \text{ and } (\overline{r \text{ or } s})$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	F
T	F	F	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

ignition on

ignition off

Why did I write this expression
with brackets?

Because

g and r or S

is ambiguous. It

could mean

$(g$ and $r)$ or S

which has a completely different
text taste!

We use the following

notation:

<u>and</u>	\wedge
<u>or</u> inclusive	\vee

Bonus exercises

1. Calculate the truth table of $(q \wedge r) \vee s$ [and observe that it is different from the one for $q \wedge (r \vee s)$].

2. Construct the truth table for the following example (use the letters p, q, r, s).

Example The audible warning for headlamps sounds if the key is removed from the ignition and the driver's door is open and either the headlamps are on or the parking lamps are on.

Draw up a truth table for this example

A = 'The audible warning for headlamps sounds'

P = 'the key is removed from the ignition'

Q = 'the driver's door is open'

R = 'the headlamps are on'

S = 'the parking lamps are on'

