Lecture 20

Revision of lectures 7 to 19

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2. (a) Define what is meant by *conjunctive normal form (CNF)* [1 mark]. Given the truth table for a wff A, explain, with reasons, how you would obtain a wff B, in CNF, which was logically equivalent to A [4 marks].

   (b) Define what is meant by a *Horn formula* [1 mark]. Write the following Horn formula, X, in *implicational form*

   $$(p \vee \neg q \vee \neg r) \wedge (\neg s \vee \neg u) \wedge (\neg p \vee \neg q \vee r) \wedge (p) \wedge (q)$$

   [2 marks]. By using the **fast algorithm** (and showing all steps), determine whether X is satisfiable or not [2 marks].

   (c) Use **truth trees** to determine whether the following is a valid argument

   $$a \to (b \to c), \neg d \vee a, b \models d \to c$$

   [5 marks].

   (d) Use **truth trees** to determine whether the following is a tautology

   $$(p \to (q \to r)) \to ((p \to q) \to (p \to r))$$

   [5 marks].

Exam continues ...

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(b) A Horn formula is a wff in CNF where each block of disjunctions contains at most one positive literal.

- $p \vee \neg q \vee \neg r \equiv \neg q \vee \neg r \vee p$
  $\equiv \neg(q \wedge r) \vee p$
  $\equiv \underline{(q \wedge r) \rightarrow p}$

- $\neg s \vee \neg u \equiv \neg(s \wedge u)$
  $\equiv \underline{\underline{(s \wedge u) \rightarrow f}}$

- $\neg p \vee \neg q \vee r \equiv \neg(p \wedge q) \vee r$
  $\equiv \underline{\underline{(p \wedge q) \rightarrow r}}$

- $p \equiv t \rightarrow p$

- $q \equiv t \rightarrow q$

The implicational form of the Horn formula is therefore

$$\left((q \wedge r) \to p\right) \wedge \left((s \wedge u) \to \underline{f}\right) \wedge \left(p \wedge q \to r\right) \wedge \left(\underline{t} \to p\right) \wedge \left(\underline{t} \to q\right)$$

Apply algorithm

p	q	r	s	u
T	T	T	F	F

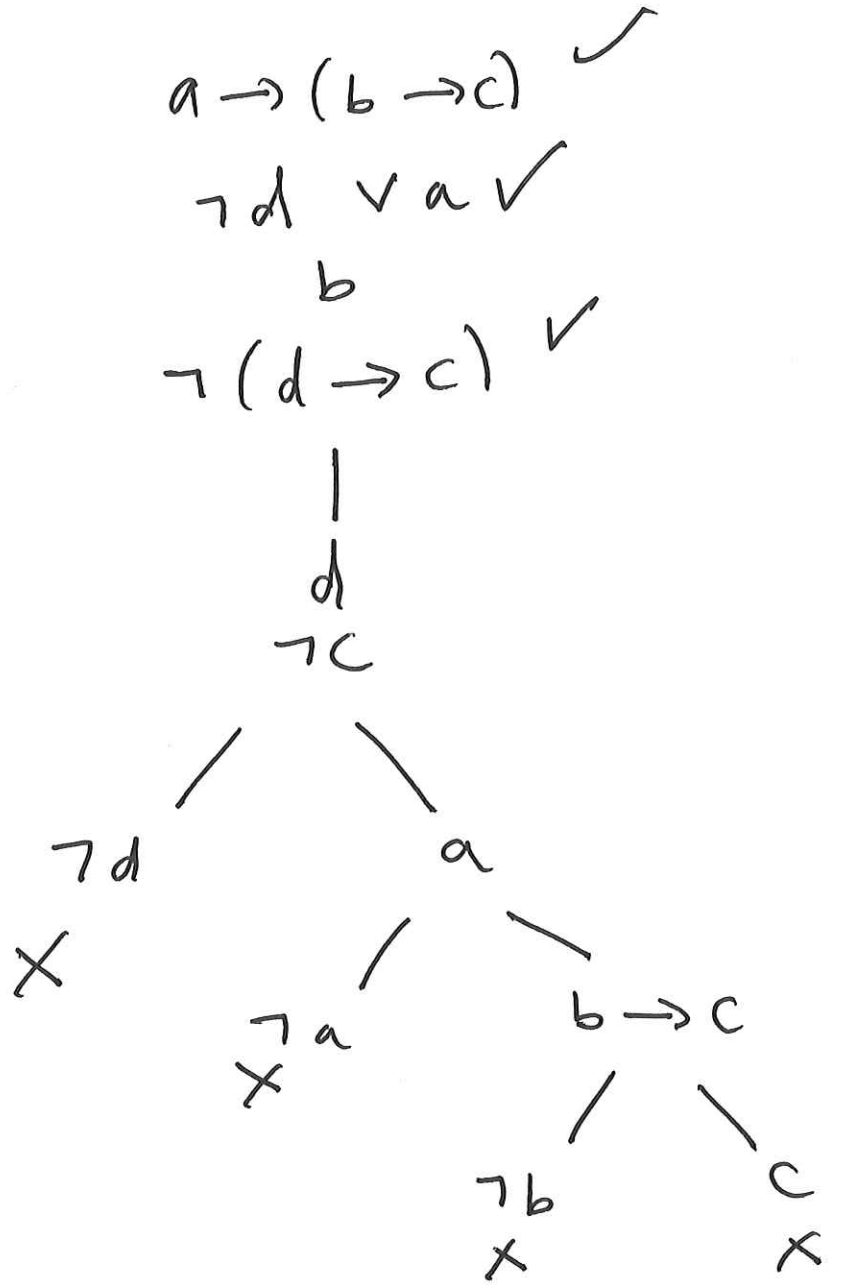
This is a satisfying truth assignment to the Horn formula.

Check

$$\overset{T}{(T} \vee F \vee F) \wedge \overset{T}{(T \vee T)} \wedge \overset{T}{(F \vee F \vee T)}$$

$$\wedge (T) \wedge (T) \equiv T$$

(c)



The truth tree closes so the argument is valid.

(d)

$$\neg[(p \to (q \to r)) \to ((p \to q) \to (p \to r))] \checkmark$$

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$$p \to (q \to r)$$

$$\neg((p \to q) \to (p \to r)) \checkmark$$

|

$$p \to q$$

$$\neg(p \to r) \checkmark$$

|

$$p$$

$$\neg r$$

/ \

$\neg p$ × | q

/ \

$\neg p$ × | $(q \to r)$

/ \

$\neg q$ × | r ×

The truth tree closes so the original wff is a tautology.