Lecture 29

Truth trees for FOL continued

Example We show that

$$(\forall x)(H(x) \to M(x)),\ H(a) \models M(a)$$

is a valid argument.

$$(\forall x)(H(x) \to M(x)) \ *$$

$$H(a)$$

NB $\longrightarrow$ $\neg M(a)$

$$H(a) \to M(a)$$

$\neg H(a)$	$M(a)$
×	×

truth tree closes so ~~[illegible]~~ argument's valid.

$$\vDash (\exists x)(A(x) \wedge B(x)) \rightarrow ((\exists x) A(x) \wedge (\exists x) B(x))$$

$A(x)$ = 'x is round'

$B(x)$ = 'x is blue'

then $(\exists x)(A(x) \wedge B(x))$ true means there is something that is round and blue.

In particular, there is a round thing and there is a blue thing. This gives us an intuitive idea as to why this sentence is universally valid.

$$\neg\left[(\exists x)(A(x) \wedge B(x)) \rightarrow (\exists x)A(x) \wedge (\exists x)B(x)\right] \checkmark$$

$$|$$

$$(\exists x)(A(x) \wedge B(x)) \checkmark$$

$$\neg\left((\exists x)A(x) \wedge (\exists x)B(x)\right) \checkmark$$

$$|$$

new name

$$A(a) \wedge B(a) \checkmark$$

$$|$$

$$A(a)$$

$$B(a)$$

$$\neg(\exists x)A(x) \checkmark \qquad\qquad \neg(\exists x)B(x) \checkmark$$

$$| \qquad\qquad |$$

$$(\forall x)\neg A(x) \qquad\qquad (\forall x)\neg B(x)$$

$$| \qquad\qquad |$$

$$\neg A(a) \qquad\qquad \neg B(a)$$

$$\times \qquad\qquad \times$$

Tree closes so sentence is universally valid.

Example Construct an interpretation

in which $(\exists x) A(x) \wedge (\exists x) B(x)$

but $(\exists x)(A(x) \wedge B(x))$ is false.

$D = \{1, 2\}$

$X = \{1\}$, $Y = \{2\}$

$A(x) \longrightarrow$ '$x \in X$'

$B(x) \longrightarrow$ '$x \in Y$'

$(\exists x)(x \in X) \wedge (\exists x)(x \in Y)$ is true.

but $\{1\} \cap \{2\} = \emptyset$ ←

$(\exists x)(A(x) \wedge B(x))$ is false.

Example: we prove that

$$(\forall x)(\forall y)A(x,y) \equiv (\forall y)(\forall x)A(x,y)$$

$$\neg\left[(\forall x)(\forall y)A(x,y) \leftrightarrow (\forall y)(\forall x)A(x,y)\right]$$

$$(\forall x)(\forall y)A(x,y) \ *$$

$$\neg(\forall y)(\forall x)A(x,y) \ \checkmark$$

$$(\exists y)(\exists x)\neg A(x,y)$$

$$\neg A(a,b)$$

$$A(a,b)$$

$$\times$$

$$\neg(\forall x)(\forall y)A(x,y)$$

$$(\forall y)(\forall x)A(x,y)$$

Δ sim.