Lecture 3

From the end of the last lecture:

Calculate the truth table of $(q \wedge r) \vee s$

q	r	s	$q \wedge r$	$(q \wedge r) \vee s$
T	T	T	T	T
T	T	F	T	T
T	F	T	F	T
T	F	F	F	F
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

Different truth table from $q \wedge (r \vee s)$.

So, these two expressions do not mean the same thing.

Example The audible warning for headlamps sounds if the key is removed from the ignition and the driver's door is open and either the headlamps are on or the parking lamps are on.

Draw up a truth table for this example

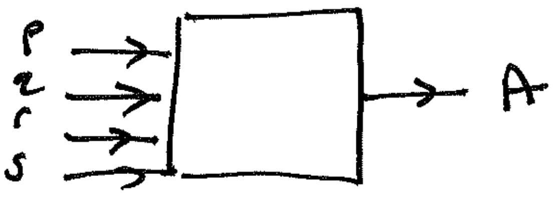
A = 'The audible warning for headlamps sounds'

p = 'the key is removed from the ignition'

q = 'the driver's door is open'

r = 'the headlamps are on'

s = 'the parking lamps are on'



$$A = (p\ \underline{\text{and}}\ q)\ \underline{\text{and}}\ \overbrace{(r\ \underline{\text{or}}\ s)}^{\text{atleast one}}$$

[I have used brackets to make reading this statement easier].

key	door	headlamps	parking lamps	
p	q	r	s	A
T	T	T	T	T ✓
T	T	T	F	T ✓
T	T	F	T	T ✓
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

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Truth tables computed using the truth table generator on the website.

q	r	s	q∧(r∨s)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

p	q	r	s	(p∧q)∧(r∨s)
T	T	T	T	T
T	T	T	F	T
T	T	F	T	T
T	T	F	F	F
T	F	T	T	F
T	F	T	F	F
T	F	F	T	F
T	F	F	F	F
F	T	T	T	F
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	F
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

Key points from those two examples

- Human language (natural language) is imprecise. For example, we need to infer from the context what 'if' means here (may mean something different elsewhere).
- Common sense tells us that here 'or' means 'at least one'. But if I say you must choose between 'logic & prog *or* statistics' then it means 'exactly one'.
- We deal with sentences that are either true or false. Such sentences are called *statements*
- statements are glued together using words like *and*, *or*, ... called *(propositional) connectives*.

I have introduced the two propositional connectives $\wedge$ (and) and $\vee$ (or-inclusive).

There are two more easy connectives

not

P	$\neg P$
T	F
F	T

not = 'it is not the case that'

xor (exclusive or)

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

We can now compute the truth values of compound statements using the above truth tables.

Examples

(1) 'Homer Simpson is PM or the Earth orbits the Sun'.

In everyday language, this is just plain odd. But we can assign it a truth value $F \vee T = T$ //

(2) (There is an elephant in the room) $\vee$ $\neg$(There is a elephant in the room) is always T, irrespective of the presence or absence of said elephant

How many propositional connectives are there? Infinitely many — but we only need a handful as we shall see later. I shall now introduce some less familiar/difficult connectives.

p implies q / If p then q.

The word 'implies' has many uses in English (many articles have been written). We have to choose one meaning for PL.

Before I define it (by means of a truth table) here is a motivating example.

<u>Example</u> Your parents make the following promise:

" <u>If</u> you pass Logic & Proof <u>then</u> they will buy you a car "

~~We are interested in~~

There are four possible situations. We shall tabulate them.

You Pass L&P	Your parents buy car	Promise Kept
T	T	T
T	F	*F*
F	T	T
F	F	T

This truth table is the basis of the truth table for our Connective →

implies

p	q	$p \to q$
T	T	T
T	F	* F *
F	T	T
F	F	T

There is one more connective we shall need.

p if and only if q usually abbreviated to ~~I~~. p iff q. Notation $p \Leftrightarrow q$.

It detects when p and q have the same truth value.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

English word	Symbol	Name
not	$\neg$	Negation
and	$\wedge$	Conjunction
or	$\vee$	disjunction
xor	$\oplus$	exclusive disjunction
implies	$\rightarrow$	Conditional
iff	$\leftrightarrow$	biconditional

Question Why these particular connectives? See later. In fact, there is a lot of flexibility in what we can choose.

Important

We can use these connectives as building blocks to construct more complex statements. The truth or falsity of these complex or **compound statements** can be calculated from the truth values of the simpler or **atomic statements** and the definitions of the connectives.

__Example__ We calculate the truth table of $A = (p \to q) \wedge (q \to p)$. The truth table will then tell us what A means.

calculations.

p	q	$p \to q$	$q \to p$	$(p \to q) \wedge (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

This is the same truth table as for $p \leftrightarrow q$

(but $p \leftrightarrow q \neq (p \to q) \wedge (q \to p)$)

α

Summary

- A *statement* is a sentence that is either true (T) or false (F). PL only deals with statements.

- The (propositional) *connectives* we shall mainly use are

$$\underbrace{\wedge, \vee, \rightarrow, \leftrightarrow, \oplus}_{\text{binary}}, \underbrace{\neg}_{\text{unary}}$$

- A statement that cannot be analysed further using these connectives is said to be *atomic*.

A compound statement is a statement constructed from atomic statements using the connectives.

- The truth or falsity of a compound statement can be calculated from the truth values of the atomic statements using the truth tables of the connectives.

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T